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THE METALLIC REFLECTION OF SELENIUM.

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Introduction.

1. *Object.*

MUCH recent work has been done on the verification of modern dispersion formulæ as applied to substances having absorption bands in the visible or infra-red regions of the spectrum. Very little has been done on substances possessing absorption bands beginning in the visible region and extending out into the ultra-violet. Since selenium is a substance of this type the determination of its dispersion curve is of interest.

The dispersion curve for wave-lengths within the absorption band can not be directly determined and recourse must be had to the indirect method of metallic reflection for determining the indices of refraction and absorption in terms of the constants of elliptical polarization, assuming some formula of connection, several of which have been developed by various investigators. This has recently been done most successfully by Pflüger for cyanin, which has a strong absorption in the green. His work shows a very satisfactory verification of the Ketteler-Helmholtz formula for dispersion and Cauchy's formulæ for metallic reflection.

The metallic reflection of selenium is of interest because it is on the border line between transparent and true metallic media, and it is from media occupying such an intermediate position that much is

to be learned concerning the true nature of metallic reflection. "Vitreous selenium" reflects blue and violet light metallically and the longer wave-lengths vitreously. By passing it over into the allotropic form known as "metallic selenium," it behaves more like a true metal for all wave-lengths, at least in the visible spectrum.

It is the object of this paper, after briefly recounting the previous work on selenium, to determine just how the reflection from vitreous selenium assumes more and more of the characteristics of true metallic reflection as the wave-length of the incident light is decreased. The phase-condition and azimuth of restored polarization of the reflected light will be studied at various incidences for three different wave-lengths. The changes in these characteristics of the reflection will be noted as the selenium is transformed from the vitreous to the metallic form. The data thus secured will be interpreted analytically in order to secure values of the indices of refraction and absorption to compare with those determined directly by other workers.

The improvements in the methods of spectrophotometry in recent years have given a very satisfactory way of testing the formulæ for the reflecting power of media whose indices of refraction and absorption are known. Murphy[1] in 1897 verified Fresnel's reflection formulæ for glass as a type of transparent media, but so far as I know, no photometric verification in the case of an absorbing medium, for various angles of incidence and various wave-lengths, has been attempted. It becomes, then, the object of the second part of this paper to verify Cauchy's formulæ (page 203) for the intensity of the light reflected from metallic media, by means of observations with a Brace spectrophotometer.

2. *Forms of Selenium.*

The properties and transformations of the allotropic forms of selenium have been treated recently by Saunders.[2] Three forms are recognized: (1) Liquid (including vitreous, amorphous and

[1] Murphy, Astro. Phys., Jan. 6, 1–10, 1897.

[2] A. P. Saunders, Jour. Phys. Chem., IV., No. 2, June, 1900, pp. 423–513. A fairly complete bibliography on selenium is included. Some references given in this paper are omitted by Saunders.

soluble selenium); (2) crystalline red, (including perhaps two closely allied forms); (3) crystalline gray or metallic. The vitreous and metallic forms alone are of interest in the present research, and since previous workers have not been uniformly clear as to which form of selenium they employed, or as to the precise treatment of their mirror surfaces, I shall state briefly the general characteristics of these two forms.

The first form has the properties of an ordinary liquid above 220° C.; below this it becomes more and more viscous, remains soft down to about 60°, and at 30°–40° gets quite hard and brittle, showing a red streak. In this form it is called vitreous selenium from its conchoidal and brilliant fracture. Amorphous selenium is the state in which the element separates from solutions of selenious acid on reduction, forming, when dry, an impalpable red powder with no trace of crystallinity. The amorphous form results whenever a deposit is secured from vaporized selenium, and the thin films of selenium secured by cathode deposit are also of the amorphous variety. Finely powdered vitreous selenium cannot be distinguished from the amorphous variety; and conversely, amorphous selenium when warmed to 40°–50° C. darkens in color and coagulates to a soft mass which, on cooling, becomes hard and brittle, assuming somewhat the fracture of the vitreous form.

Gray crystalline or metallic selenium is obtained from any of the other forms at higher temperatures; in presence of certain liquids, the change takes place even at ordinary temperatures. This is the stable form of selenium at all temperatures from ordinary temperatures to the melting point. The other forms are unstable, the red crystalline representing an intermediate stage between "liquid" and metallic selenium. The reverse transformation from metallic into any other form is not possible below 217° C. With lapse of time the vitreous form passes gradually over into the gray crystalline or metallic form. The change in the optical properties accompanying this transition is illustrated by the observations on fresh and old mirrors.

While the preliminary and check observations in this research were made with mirrors of commercial selenium, which is the vitreous modification and contains considerable sulphur and some

tellurium as impurities, the mirrors for which the final values are given were made of chemically pure vitreous selenium. Some of the mirrors were converted into the metallic form in a manner described later.

3. *Optical Properties of Selenium. Emission and Absorption Spectra.*

Historical. — The previous work on the optical properties of selenium has comprised observations of the emission and absorption spectra of selenium vapor, the constants of elliptical polarization of light reflected from selenium mirrors, and the determination of its refractive index directly and indirectly. Recently, while this research was under way, Nutting has published values of the reflection coefficient for different wave-lengths.

In 1863 Werther[1] noted the similarity between the spectra of selenium and lead. Ditte[2] (1871) found well-marked yellowish-green lines in the selenium spectrum toward the *E* line, and also two brilliant blue bands near the *G* line. The work of Salet and of Plücker and Hittorf[3] is summarized in the British Association Reports for 1884. The series spectra of selenium, showing principally triplets, was investigated by Runge and Paschen[4] in 1897 and its analogy with the series spectra of oxygen and sulphur was noted.

The absorption spectrum of selenium vapor was studied by Salet (1871) and Gernez (1872), and it was found that white light passed through a porcelain tube containing selenium heated in an atmosphere of dry carbonic anhydride is gradually absorbed from the violet up to the red, without any appearance of black lines; on raising the temperature considerably the spectrum becomes brighter and dark bands appear furrowing the blue and violet.

HISTORICAL ACCOUNT OF THE WORK ON METALLIC REFLECTION OF SELENIUM.

The earliest work on the metallic reflection of selenium, as far as I have found, is that of Jamin[5] (1850), who records the

[1] Werther, J. pr. Chem., 88, pp. 180–181, 1863.

[2] Ditte, Comp. Rend., 73, p. 622, 1871; Chem. Soc. Jour. (2), 9, p. 1146, 1871.

[3] Brit. Assoc. Rep., 1884, pp. 440, 441.

[4] Runge and Paschen, Wied. Ann. 61, p. 641–686, 1897; also Astro. Phys. Jour., 8, pp. 70–101, 1898.

[5] Jamin, Ann. de Chim. d. Phys., 29, p. 303, 1850.

value of the principal incidence, the coefficient of ellipticity and the ratio of amplitudes of the principal incidence, for a single wave-length.

Quincke (1874) determined the principal incidence and principal azimuth for a number of metals, including selenium, by a modification of Brewster's double mirror method.[1] Though not so stated, his mirrors were in all probability of vitreous selenium. Just before observation they were polished with buckskin and a pinch of jeweler's rouge (p. 342, loc. cit.). Quincke's values of the principal azimuth are much higher than those of other observers, and the dispersion curve deduced from his results by Ketteler[1] gives a steadily *rising* refractive index (n) with increasing wave-length, which is contrary to all other observations, to be discussed presently. Some experiments looking toward a possible explanation of this discrepancy will be mentioned later on.

Glan[2] in 1879 studied the change in phase of monochromatic red light upon reflection from selenium surfaces by the method of Newton's rings for their films. He used vitreous selenium melted between glass plates. His angles of incidence ranged from 18° to 70°, and his results show no great difference for red light between selenium and ordinary transparent substances. He also determined the variation in the difference of phase (Δ) with changing wave-length (λ) (for a given incidence of 30°); his results show only a small change in Δ with change in λ, but Δ increases with decreasing λ, as does the absorption of selenium.

Using a Babinet compensator in the ordinary way Glan measured the phase-difference for selenium in the neigborhood of the principal incidence (φ) and determined φ by interpolation for wave-lengths 631$\mu\mu$ and 508$\mu\mu$.

The latest work on the metallic reflection of selenium is that of Cornu[3] (1889) who investigated a portion of the ultra-violet as well as the visible spectrum. In the visible region the phase condition of the reflected light was determined in the usual way with a Bravais compensator.

[1] This method is well described and its accuracy shown in a paper by Pflüger on the Metallic Reflection of Solid Cyanin, Wied. Ann., 65, pp. 214–224, 1898.

[2] Ketteler, "Theoretische Optik," Braunschweig, pp. 548–552, 1885.

[3] Glan, Wied. Ann., 7, pp. 650–652, 1879.

[4] Cornu, Comp. Rend., CXIII., 1889, pp. 917–923, 1211.

In the ultra-violet region fluorspar lenses were used and approximate adjustments were made by means of a fluorescent eye-piece the final observations being secured photographically. The difficulty of effecting the double operation of compensator and analyzer was eliminated by using a Jamin-Babinet compensator and noting the displacement of the fringes instead of moving the quartz wedge. The reflected light was focused directly on the face of the compensator, which bore a ruled line for reference; an image of this, together with that of the displaced fringes, was focused by a second fluorspar lens on a sensitive plate, between which and the compensator was placed the analyzing nicol. Since the position of the fringes is independent of the azimuth of the analyzer, a single exposure would suffice to determine the fringe-displacement, if

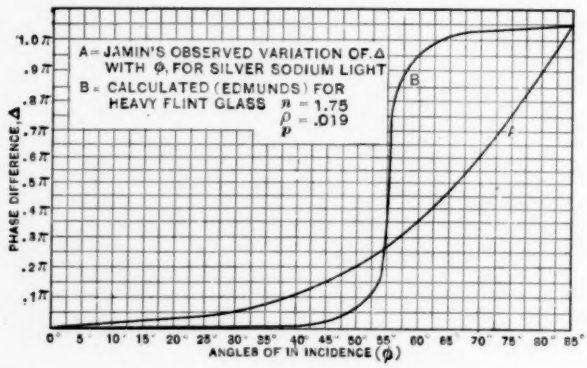
A = JAMIN'S OBSERVED VARIATION OF Δ WITH φ, FOR SILVER SODIUM LIGHT
B = CALCULATED (EDMUNDS) FOR HEAVY FLINT GLASS n = 1.75 ρ = .019
B
A
PHASE DIFFERENCE, Δ
1.0π .9π .8π .7π .6π .5π .4π .3π .2π .1π
0° 5° 10° 15° 20° 25° 30° 35° 40° 45° 50° 55° 60° 65° 70° 75° 80° 85°
ANGLES OF IN INCIDENCE (φ)

Fig. 1.

the analyzer is in position to give the bands at all. The exact azimuth of the reduced polarization was determined by a series of eight or ten exposures on the same plate, the analyzer being rotated each time through a small known angle; the azimuth for which there was maximum intensity of the fringes is the desired principal azimuth, when the angle of incidence corresponds to a fringe-displacement of $\pi/2$. This method is of course less precise than the measurements in the visible spectrum; but, so far as I know, this is the only recorded attempt to determine directly the phase relations in metallically reflected ultra-violet light.

Cornu's[1] mirrors were of vitreous selenium melted and pressed

[1] Cornu, Comp. Rend., LXXXVI., p. 649, 1878.

out between surfaces of polished quartz. He used two mirrors, one freshly made and the other seven years old. Age gave a more metallic character to the reflection, increasing the absorption very noticeably. Additional evidence along this line was secured in the present research.

The results of all previous workers are given in the accompanying table.[1] The wide variation in their observations exhibits the desirability of further work along this line. The work of Glan and of Cornu showed that selenium behaves as a true metal only for the shorter wave-lengths, acting more like a transparent substance for the longer wave-lengths. The difference between the forms of the curves in which the phase-difference (Δ) is plotted as a function of the incidence (φ) for vitreous and for metallic reflection was pointed out by Cornu in 1878, and is shown in the accompanying curves, Fig. 1, for glass and silver. The former shows an inflection at the point $\Delta = \pi/2$ (*i. e.* $\lambda/4$) so that the

TABLE I.

Previous Values of Constants of Elliptical Polarization of Light Reflected from Selenium Surfaces.

λ	$\bar{\phi}$	$\bar{\psi}$	Observer.	References.
	68°10′	10°10′		
280$\mu\mu$	(64 00)	(11 12)	Cornu.	
431	70 10	15 30	Quincke.	*Jamin:* 1850, Ann. d. Chim. et Phys., Vol. 29, p. 303.
439	71 30	5 00		
439	(68 15)	(7 58)	Cornu.	*Quincke:* 1874, Pogg. Ann., Jubel Bd., p. 336.
486	71 00	13 14	Quincke.	
508	71 28	7 31	Glan.	*Glan:* 1879, Wied. Ann., Vol. 7, p. 640.
527	71 10	12 13	Quincke.	
589	71 23	11 32	Quincke.	
589	68 05	9 55	Jamin.	*Cornu:* 1889, Comptes Rendus, Vol. CVIII., p. 917, 1211.
631	69 49	4 8	Glan.	
656	71 26	8 24	Quincke.	
721	69 35	0 30	Cornu.	
721	(69 00)	(7 00)		

Cornu used two mirrors, one fresh, another seven years old. The figures in parentheses are for the old mirror.

[1] Plotted from the results given by Jamin, Ann. de Chim. et d. Phys., 19, p. 315, 1847.

rate of change of phase-difference with respect to the angle of incidence is very large in this region, whereas for metallically reflecting media, the phase-difference rises steadily with increasing incidence, and shows no such inflection. Such a curve showing the connection between the phase difference and the angle of incidence for any one wave-length can not be secured from the observations already cited, and so it was the object of the first part of the present investigation to secure such curves for several wave-lengths which will show just how the reflection from selenium surfaces acquires more or less of a true metallic character as the wave-length of the incident light varies. The general conclusion is that there is no sharp distinction between the two cases of vitreous and metallic reflection, and that selenium, like several other substances such as the aniline dyes, presents the transition from one to the other in a continuous manner as the wave-length of the incident light is changed.

PRESENT WORK ON THE REFLECTION OF LIGHT FROM SELENIUM.

Introduction.

The present experimental work on the reflection from selenium surfaces is divided into two main parts in the visible spectrum: (1) The determination of the phase condition and the azimuth of restored polarization of the reflected ray for different angles of incidence for red, yellow and blue light; (2) the direct observation of the reflecting power for various wave-lengths and at different angles of incidence by means of a spectrophotometer.

From a knowledge of the constants of the elliptical polarization of the reflected light as secured from part (1), the values of the refractive index (n) and the absorption coefficient (k') will be secured by an analytical interpretation of the phenomena, and then compared with the directly determined values of k' and n secured by other observers.

The directly observed values of the reflecting power R, secured from part (2) and those of previous workers will be compared with the values calculated from k' and n as given by part (1).

The foundations of the theory of metallic reflection are far from satisfactory physically, and accumulated experimentation under

proper and varied conditions is needed for an advance in this direction. The formulæ here used in the reduction of observed quantities to give the optical constants of the medium are not to be regarded as constituting a real theory of metallic reflection, but rather as convenient mathematical aids which, though founded on an unsatisfactory physical basis, have been found to give results which agree well with experiment. A theory of metallic reflection phenomena is yet to be developed in terms of the ionic theory of matter.

Metallic Reflection Formulæ.

Before recounting the present experimental work on the metallic reflection of selenium, attention is called to the formulæ which have been employed throughout the investigation in the reduction or interpretation of experimental results.

Notation.

$n_i \equiv$ index of refraction for incidence i.

$n_0 \equiv$ index of refraction for normal incidence; when written simply n, the variation with incidence is not considered.

$k \equiv$ absorption coefficient as defined by Drude.

$k' \equiv nk$, *i. e.*, absorption coefficient as defined by Walter; see p. 203.

$k_i' \equiv$ absorption coefficient for incidence i.

$k_o' \equiv$ absorption coefficient for incidence 0°, when written simply k', no account is taken of its variation with changing incidence.

$\Delta \equiv \delta_p - \delta_s \equiv$ difference in phase between the two components of the elliptically polarized reflected light, polarized in and normal to the plane of incidence.

$\psi \equiv$ azimuth of "restored" or "reduced" polarization of the reflected light with reference to the plane of incidence.

$\tan \psi = \rho \equiv \dfrac{A_p}{A_s} =$ ratio of the amplitudes of the two components of the reflected light.

$\varphi \equiv$ angle of incidence.

$\bar{\varphi} = \varphi$ and $\bar{\psi} = \psi$ when $\Delta = \pi/2$, *i. e.*, $\bar{\varphi}$ and $\bar{\psi}$ are the "principal incidence" and "principal azimuth."

$$R \equiv \text{reflecting power} \equiv \frac{\text{intensity of reflected light}}{\text{intensity of incident light}}$$
$$= \frac{A_R^2}{A\varphi^2}, \text{ for normal incidence.}$$

$R_i^p \equiv$ reflecting power at incidence i for light plane polarized in the plane of incidence.

$R_i^s \equiv$ same for light polarized normal to the plane of incidence.

Formulæ.

Approximate.[1]

$$(a) \qquad \begin{cases} k = \sin \Delta \tan 2\psi \\ n = \sin \varphi \tan \varphi \cdot \dfrac{\cos 2\psi}{1 + \cos \Delta \sin 2\psi} \\ n^2(1 + k^2) = \sin^2 \varphi \tan^2 \varphi \cdot \dfrac{1 - \cos \Delta \sin 2\psi}{1 + \cos \Delta \sin 2\psi} \end{cases}$$

$$(a') \qquad \begin{cases} k = \tan 2\bar{\psi}, & \text{At principal incidence or} \\ n = \sin \bar{\varphi} \tan \bar{\varphi} \cos 2\bar{\psi} & \Delta = \pi/2. \end{cases}$$

More rigorous.[1]

Putting

$$\sin \Delta \tan 2\psi = \tan Q, \quad \cos \Delta \sin 2\psi = \cos P,$$
$$\tan \tfrac{1}{2}P \sin \varphi \tan \varphi = S,$$

and neglecting $1/S^4$ in comparison with 1,

$$(b) \qquad \begin{cases} n = S \cos Q\left(1 + \tfrac{1}{2}\dfrac{\sin^2 \varphi}{S^2}\right) = S \cos Q(1 + V) \\ nk \equiv k' = S \sin Q(1 - V), \quad V \equiv \tfrac{1}{2}\dfrac{\sin^2 \varphi}{S^2} \\ k = \tan Q\left(\dfrac{1 - V}{1 + V}\right) \text{ or } = \tan Q\left(1 - \dfrac{\sin^2 \varphi}{S^2}\right). \end{cases}$$

(If we further neglect $1/S^2$ in comparison with 1, we get from these last equations those under (a) above.)

At principal incidence:

$$\tan \varphi = \tan 2\bar{\psi}, \quad \cos P = 0, \quad S = \sin \bar{\varphi} \tan \bar{\varphi}.$$

[1] Drude: Theory of Optics, Amer. ed., pp. 274–295, 358–368; Wied. Ann., No. 36, p. 544, 1889; No. 39, p. 507, 1890; No. 64, pp. 161, 162, 1898. See also Winkelmann, Handbuch d. Phys., p. 826.

Whence

$$(b') \qquad \begin{cases} n = \sin\bar{\varphi}\tan\bar{\varphi}\cos 2\bar{\psi}\left(1+\dfrac{\cot^2\bar{\varphi}}{2}\right) \\ k' \equiv nk = \sin\bar{\varphi}\tan\bar{\varphi}\sin 2\bar{\psi}\left(1-\dfrac{\cot^2\bar{\varphi}}{2}\right) \\ k = \tan 2\bar{\psi}\,(1-\cot^2\bar{\varphi}). \end{cases}$$

Taking the variation of k' and n with φ into account, Ketteler[1] has deduced the equations of metallic reflection giving n and k' in terms of Δ, φ and φ, which Walter[2] has put in the following form:

More Rigorous Formulæ in a Different Form.

Putting

$$\sin\bar{\varphi}\sin 2\bar{\psi} = \sin\chi,$$
$$\sin 2\bar{\varphi}\sin 2\bar{\psi} = \sin\theta,$$
$$k'_{\bar{\Phi}} = \tan\bar{\varphi}\sin\chi,$$
$$k' \equiv nk.$$
$$n_{\bar{\Phi}} = \tan\bar{\varphi}\cos\chi,$$

$$(c) \qquad \begin{cases} k'_{\Phi=0} \equiv k_0' = \tan\bar{\varphi}\surd\sin(\chi+\theta/2)\sin(\chi-\theta/2), \\ n_0 = \tan\bar{\varphi}\surd\cos(\chi+\theta/2)\cos(\chi-\theta/2). \end{cases}$$

Formulæ for Reflecting Power. (*Cauchy.*)

For normal incidence, *i. e.*, $\varphi = 0$.

Light polarized in any azimuth:

$$(d) \qquad R = \frac{n_0^2(1+k_0^2)+1-2n_0}{n_0^2(1+k_0^2)+1+2n_0} = \frac{(n_0-1)^2+k_0'^2}{(n_0+1)^2+k_0'^2},$$

at any angle of incidence $\varphi = i$. For light polarized *in* the plane of incidence,

$$(e) \qquad R_i^p = \frac{\sin^2(i-r_i)+k_i'^2\sin r_i}{\sin^2(i+r_i)+k_i'^2\sin r_i};$$

and for light polarized *normal* to the plane of incidence,

$$(e) \qquad R_i^s = \frac{\cos^2(i+r_i)\tan^2 i+k_i^2\sin^2 r_i}{\cos^2(i-r_i)\tan^2 i+k_i^2\sin^2 r_i}\cdot R_i^p;$$

[1] Ketteler, Theoretische Optik, Braunschweig, 1885.

[2] Walter, "Die Oberflächen oder Schiller-Farben," Braunschweig, 1895. See also Pflüger, Wied. Ann., LXV., p. 220, 1898.

and $R = \frac{1}{2}(R_i^p + R_i^s)$, for ordinary light.

In these equations $\sin r_i = \dfrac{\sin i}{n_i}$.

$$n_i = \left[\tfrac{1}{2}\left\{\sin^2 i + n_0^2 - k_0^2 + \sqrt{(\sin^2 i - n_0^2 + k_0'^2) + 4n_0^2 k_0'^2}\right\}\right]^{\frac{1}{2}}$$

$$k_i' = \left[\tfrac{1}{2}\left\{\sin^2 i - n_0^2 + k_0^2 + \sqrt{(\sin^2 i - n_0^2 + k_0'^2) + 4n_0^2 k_0'^2}\right\}\right]^{\frac{1}{2}}$$

or

$$(f) \qquad \begin{cases} n_i = \dfrac{1}{\sqrt{2}}(\sin^2 i + a + b)^{\frac{1}{2}}, \\ k_i' = \dfrac{1}{\sqrt{2}}(\sin^2 i - a + b)^{\frac{1}{2}}, \end{cases}$$

where $a = n_0^2 - k_0'$ and $b = \{(\sin^2 i - a)^2 - 4n_0^2 k_0'^2\}^{\frac{1}{2}}$.

A very essential difference in notation arises between these formulæ, the failure to notice which has led to errors in some recent papers on metallic reflection. Ketteler and Walter use k to represent the absorption coefficient of the medium in question for a given wave-length of light, regarding it as the damping factor per *wave-length measured in the ether*. On the other hand, Drude uses k to represent the damping factor per wave-length *measured in the medium itself*. The k of Walter is greater than the k of Drude in the ratio of the index of refraction n of the medium, or $k' = nk$, calling k of Walter $\equiv k'$, as is done throughout this paper.

The Constants of Metallic Reflection.

(*a*) *Apparatus and Method of Observation.*

A Meyerstein spectrometer furnished with Nicol's prisms on collimator and telescope (lenses: 25 mm. diameter, 20 cm. focal length, Nicols: 22 mm. × 16 mm. aperture) and with a Jamin-Babinet compensator, was used in the ordinary way to determine the phase difference (Δ) between the two components of the elliptically polarized reflected light in and normal to the plane of incidence, and the azimuth of restored polarization. The shift of the compensator bands was determined with reference to a pair of fixed parallel wires mounted directly on the face of the stationary wedge, and slightly farther apart than the width of a band. The linear scale on the compensator was graduated to .5 mm., and the divided head carried

by the screw operating the movable wedge was graduated in 100 parts so that .005 mm. could be measured directly. The distance between two consecutive bands for the shortest wave-length (432 $\mu\mu$) used was 16.78, so that the instrument allowed a difference in phase of .0003 of a wave-length to be measured provided the sharpness of the bands permited so close a setting. As a matter of fact the error in setting on the bands was such that the mean of ten readings is sure only to about .001 λ. Ten settings were made on each band with the screw always moving right-handedly. The variation from the mean is less than 1 per cent. and the variation in the determinations of the distances between consecutive bands is about 0.3 per cent. Settings were made on the bands with the analyzer in each of the conjugate positions, and from three to five bands were observed in each case in order to eliminate any variations in the figure of the wedges. A preliminary test of the wedge and a calibration of the screw proved the combination to be reliable to within the range of error of setting on a band.

In the azimuth determinations twenty settings of the analyzer were made for each of the conjugate positions, making forty settings in all for the full determination of an azimuth. Ten readings were taken in each quadrant in order to eliminate any eccentricity of the nicol in its mounting and also any local errors of graduation on the circular head, which could be read by a vernier to 1′ of arc. Successive settings for maximum sharpness of the compensator bands varied by about three degrees, generally less, and the probable error in the azimuth determination, so far as the purely instrumental errors are concerned, is about 25′.[2]

The position of the central table carrying the mirror could be read directly to 1′ and the position of this table when the mirror was normal to the collimator could be determined by successive trials to within 2′. The angle of incidence is then accurate to be about this amount. The telescope was brought into the proper position by means of a vertical hair in the eye-piece which was brought to the center of the reflected image of the collimator slit

[1] See Drude, Theory of Optics, pp. 255–258; or Preston, Theory of Light, pp. 404–409.

[2] The effect of a poor mirror surface on the value of the azimuth is noted later on.

for each new angle of incidence, and was then focused on the compensator. Since both the bands and the reference wires are outside the telescope, changing the focus back and forth from the collimator slit to the face of the compensator introduced no error.

The source of light for the red and blue was a Nernst filament (110 volts) the radiation from which was focused on the collimator slit after passing through appropriate screens to render it monochromatic. A plate of dense cobalt glass combined with a potassium bichromate screen gave a bright nearly monochromatic band of red light, and in the blue a double thickness of cobalt glass combined with a thin sheet of "signal green" gave a fairly monochromatic band. The wave-lengths transmitted by these screens were determined by locating the maximum intensity of the light transmitted by them when placed before the slit of an auxiliary prism-spectrometer, the horizontal surface of which had been previously calibrated in terms of known Fraunhofer lines. These wave-lengths were 735 $\mu\mu$ and 432 $\mu\mu$ respectively. For the yellow, a sodium flame was, used. The correctness of the wave-length determinations is tested by their agreement with the distances between consecutive compensator bands for the three different colors.

	Red. Sc. Divs.	Yellow.	Blue.
Distance between bands.	28.56	22.86	16.78
Wave lengths.	735 $\mu\mu$.	589 $\mu\mu$.	432 $\mu\mu$.
Ratio.	.03885	.03881	.03861

(*b*) *Method of Making Mirrors.*

(b_1) *Vitreous Selenium.* — To give accurate results the mirror surfaces must be free from scratches, free from films of any foreign substances, and must be perfectly plane; the latter is a rigorous requirement only in measurements of the absolute reflecting power.

In order to secure as clean mirrors as possible, the selenium was melted in a small porcelain crucible and a portion of the melted substance from beneath the surface was drawn up by suction into a hot glass tube; from this a bead of molten selenium about 5 mm. in diameter was blown upon a clean warm glass plate and after being pressed out into a circular film some 2 cm. in diameter between this

and a second warm glass plate, was allowed to cool under moderate pressure. When cold, one of the glass plates was knocked off. From a large number of mirrors thus made, the best were selected.

The glass covers were of the plane parallel "crystal" plate, selected for freedom from scratches. They were thoroughly cleaned, and just before being warmed to receive the bead of molten selenium, they were wiped with clean cotton wool dampened with alcohol and then wiped dry. With these precautions and using c. p. selenium, good mirror surfaces could be secured, without polishing, of about two to five square centimeters area. Any small regions which seemed defective were covered with black paper, as were also all exposed parts of the glass backing.[1] When not actually in use, the mirrors were wrapped in clean tissue paper. These details are given because the treatment and condition of the mirror surfaces are of prime importance on account of the rôle which the so-called "surface-layer" plays in metallic reflection.

(b_2) *Metallic Selenium.* — Some of the films of vitreous selenium thus formed were converted before the glass plate was removed into the metallic or gray crystalline variety by being heated in an oven at about 180° C. for 4–6 hours, and then allowed to cool gradually. When cold they no longer adhered to the glass covers and were very brittle; their surfaces were far from mirror-like, showing a dull gray luster. To secure a mirror such a film was cemented to a glass plate and the exposed surface thoroughly polished with soft chamois skin and jeweler's rouge, and just before observation they were rubbed slightly with clean chamois. An attempt was made to get a better surface before polishing with the rouge by rubbing the metallic film with fine "crocus" cloth, but the selenium scratched too readily. The mirrors of metallic selenium were not so perfect as the vitreous ones, and the numerical values given for them later on are to be regarded merely as indicating the direction of the modification in the reflected light, not the exact magnitude of the change.

The effect of polishing a mirror of vitreous selenium was also tried in order to correlate, if possible, Quincke's values for polished mirrors with Cornu's and my own for unpolished vitreous mirrors.

[1] Cf. Drude, Wied. Ann., 39, p. 497, 1890.

TABLE II.

Kind of Mirror: Vitreous Selenium, Unpolished.

Wave-Length: 735μμ. Distance Between Bands: 2a = 28.56 Sc. Div.					589μμ 22.86.				432μμ 16.78.			
ϕ	Shift δx	Phase Δ in π	ψ	Ratio of Amplitudes A_p/A_s	δx	Δ	ψ	A_p/A_s	δx	Δ	ψ	A_p/A_s
20°			41°21′	.880	0.60	.0525	42°08′	.905	0.38	.0453	42°03′	.902
30	0.32	.0224					40 36	.857				
40			32 14	.630	1.60	.1400	34 09	.678	0.61	.0727	32 19	.633
50							27 15	.515				
60	0.87	.0610	16 51	.303	2.30	.2012	18 25	.333	1.18	.1405	23 53	.443
65	1.32	.0926	10 24	.184	2.98	.2607	13 30	.240	1.88	.2241	13 36	.242
67			8 50	.155								
68	3.72	.2609	7 45	.136	3.40	.2974	11 42	.207	2.42	.2884	11 33	.204
69												
70	6.88	.4825	6 17	.110	4.52	.3954	9 03	.159	3.14	.3742	10 09	.179
71												
72												
73	10.91	.7651	8 14	.145	7.64	.6684	9 34	.168	4.76	.5673	10 55	
75	12.54	.8794	11 33	.204	8.66	.7576	10 35	.187	5.31	.6329	12 00	.213
78	13.11	.9194	16 55	.304								
80	13.48	.9453	22 35	.416	10.46	.9151	19 43	.358	6.90	.8224	17 43	.320

Notation: $\delta x \equiv$ mean shift of bands with reference to their position for direct transmission. $\Delta \equiv \delta_p - \delta_s = \frac{\delta x}{a}\pi$ where $a =$ half distance between consecutive bands. $\psi = \frac{\psi'' - \psi'}{2}$ where ψ'' and ψ' are the mean settings of the analyzer for the two sets of bands.

$$\frac{A_p}{A_s} = \frac{\text{amplitude } /\!/ \text{ to incidence-plane}}{\text{amplitude } \perp \text{ to incidence-plane}} = \tan \psi.$$

(*e*) *Results.*

(c_1) *General Transition from Vitreous to Metallic Reflection with Decreasing Wave-length.* — The observed values of the phase-difference (Δ) and the azimuth (ψ) for various angles of incidence for different wave-lengths and for the several kinds of mirrors are given in the following tables (II.–IV.) and are also represented graphically in the accompanying curves.

The quantities observed for each incidence are: (1) The settings of the analyzer for maximum sharpness of the two sets of compensator bands, from which the azimuth is secured by the relation $\psi = \frac{1}{2}(\psi'' - \psi')$; (2) the position of the bands in each case, from which, by reference to their positions for direct transmission, δx

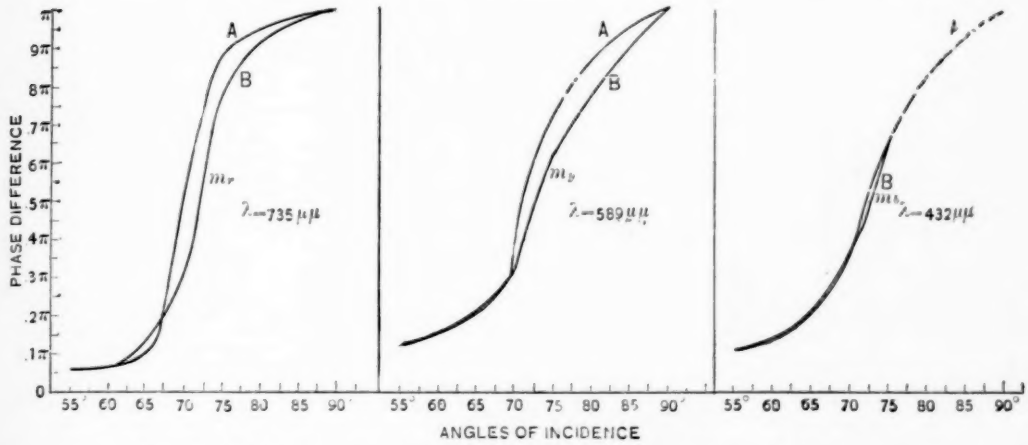

Fig. 2. Difference in the phase conditions of light reflected from mirrors of vitreous and metallic selenium. Curves *A* are for vitreous selenium. Curves *B* are for metallic selenium.

can be determined, and so Δ by the relation $\Delta = \pi\delta x/a$, where a is the half-distance between consecutive bands. The ratio of amplitudes A_p/A_s is given by the relation $A_p/A_s = \tan\psi$.

Table II. is for an unpolished mirror of vitreous selenium. The variation of phase-difference and of amplitude-ratio with change of incidence is shown by the curves in Figs. 2, 3 and 5, from which it is seen by comparison with those of Fig. 1 for glass and for silver, that selenium behaves in the matter of elliptical polarization by reflection more like a transparent substance for the longer wave-

lengths, and more metallically for the shorter wave-lengths. The gradual disappearance of the region of inflection near $\Delta = \pi/2$ as we pass from red to blue is well brought out in the phase curves,

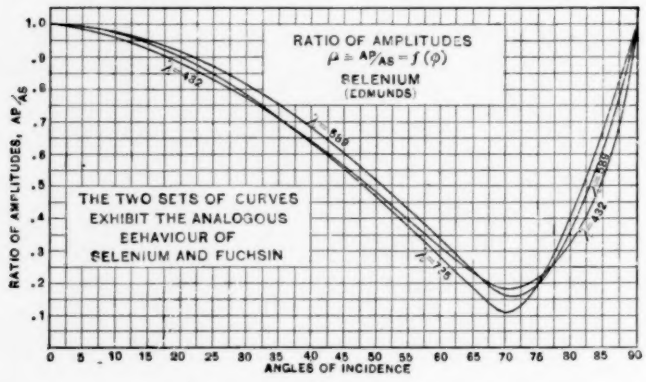
RATIO OF AMPLITUDES
$\rho = A_P/A_S = f(\varphi)$
SELENIUM
(EDMUNDS)
THE TWO SETS OF CURVES EXHIBIT THE ANALOGOUS BEHAVIOUR OF SELENIUM AND FUCHSIN
RATIO OF AMPLITUDES, A_P/A_S
ANGLES OF INCIDENCE

Fig. 3.

as is also the disappearance of the sharpness of the minimum in A_p/A_s in the amplitude curves.

Similar curves secured by Merkel for fuchsin, Figs. 4 and 6, are presented for comparison with those of selenium. The change in the character of the phase curves and of the amplitude curves, as

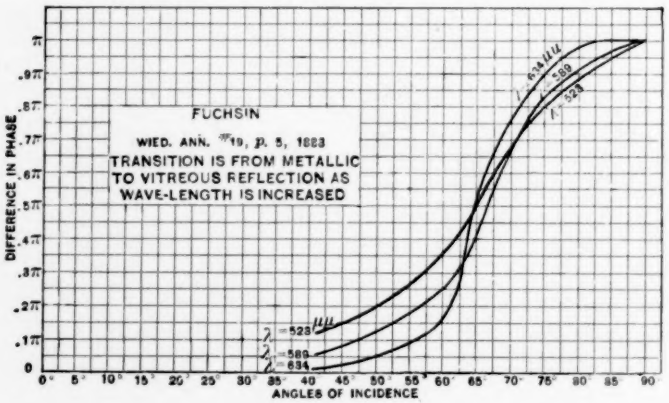
FUCHSIN
WIED. ANN. 19, p. 5, 1883
TRANSITION IS FROM METALLIC TO VITREOUS REFLECTION AS WAVE-LENGTH IS INCREASED
DIFFERENCE IN PHASE
ANGLES OF INCIDENCE

Fig. 4.

we pass from longer to shorter wave-lengths, is perfectly analogous in the two cases, though, of course, for fuchsin the amount or rapidity of the transition is much more marked than for selenium.

This gradual disappearance of the region of inflection in the phase curves and the broadening or rounding off of the minimum in the amplitude curves is characteristic of substances possessing surface color, and the magnitude of the transition as different wave-lengths

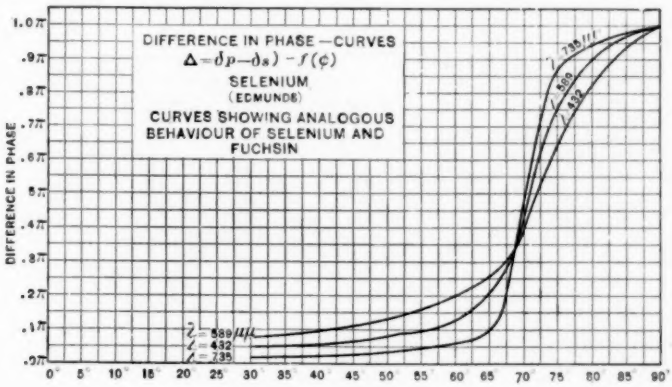

Fig. 5.

are used determines the prominence or intensity of the surface color. I shall discuss the surface color of selenium in more detail later in the paper.

Curves *A* and *B* in Fig. 1 represent the two limiting cases of

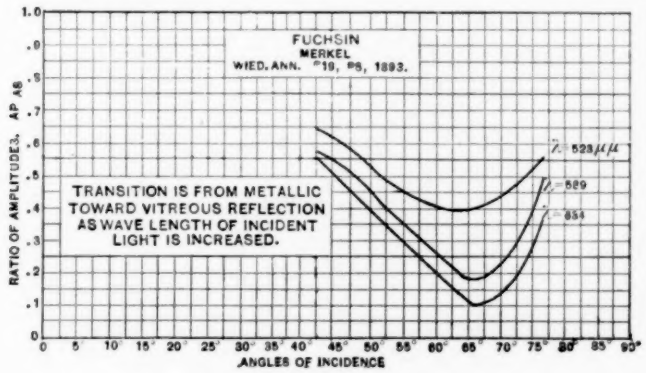
FUCHSIN
MERKEL
WIED. ANN. 19, 95, 1893.
TRANSITION IS FROM METALLIC TOWARD VITREOUS REFLECTION AS WAVE LENGTH OF INCIDENT LIGHT IS INCREASED.
RATIO OF AMPLITUDES, AP AS
ANGLES OF INCIDENCE
$\lambda = 523\mu\mu$
$\lambda = 589$
$\lambda = 854$

Fig. 6.

elliptical polarization by reflection; the first is characteristic for very heavily absorbing media, and the latter for perfectly transparent media. The equations which can be deduced on the electromagnetic theory for a relation between the difference in phase

and the angle of incidence in terms of the optical constants of the medium apply perfectly well to these limiting cases. But in the case of a medium which is neither transparent nor very heavily absorbing, such as selenium or fuchsin for certain wave-lengths, notably the red, some ambiguity may arise as to which set of equations to apply, *i. e.*, under which method of interpretation shall the experimental results be treated, for the experimental curves themselves lie between the two characteristic curves A and B. An examination of the analytical development by which the final relations between φ, Δ, ψ, k' and n are obtained in the case of true metallic reflection seems to show that there is no objection to applying them to any case where k' has a value, no matter if small; and this has been done by Pflüger[1] and others in deducing k' and n from observed values of the principal incidence $\bar{\varphi}$ and the principal azimuth $\bar{\psi}$ in the case of fuchsin and cyanin, the phase curves of which are intermediate between A and B (Fig. 1) as are the curves for selenium. Ketteler, in giving an analytical interpretation of Quincke's observations on selenium, applied the equations of metallic reflection, but in the case of fuchsin he applied those of vitreous reflection also and presented the comparison of the two methods of interpretation as applied to this latter substance.

This comparison in the case of selenium is presented in the following:

(*a*) Using the experimental curves giving Δ as a function of φ and determining that incidence $\varphi = \bar{\varphi}$ for which $\Delta = \pi/2$, by means of Lagrange's interpolation formula, we get

For $\lambda =$	735 $\mu\mu$	589 $\mu\mu$	432 $\mu\mu$
$\varphi_{\Delta=\pi/2} =$	70° 08′	71° 06′	71° 34′
A_p/A_s	.112	.156	.182
$\bar{\psi} = \tan_{\Delta=\pi/2}^{-1}(A_p/A_s)$	6° 23′	8° 55′	10° 20′

These values show an increase in both principal incidence and principal azimuth with decreasing wave-lengths; which is in agreement with the results of Cornu and Glan, but contrary to those of Quincke. The value of $\bar{\varphi}$ determined directly with a quarter-wave plate for sodium light was $\bar{\varphi}_D = 71° 32'$, and $\bar{\psi}_D = 9° 15'$. Quincke found by the method of double reflection $\bar{\varphi}_D = 71° 23'$, but his

[1] Pflüger, Wied. Ann., 5, 1898, pp. 192–224.

value of $\overline{\psi}_D^1$ is high. I adopt as the final value of φ for $\lambda = 589\mu\mu$, the mean of my two determinations, viz: $\overline{\psi} = 71^\circ\ 19'$.

(*b*) Interpreting the experimental curves of vitreous selenium as those of a transparent substance as opposed to a metallically reflecting surface, we can by interpolation determine the angle of maximum polarization, the tangent of which is the index of refraction. We get:

	$\lambda = 735\ \mu\mu$	$589\ \mu\mu$	$432\ \mu\mu$
ρ_p	.1095	.1509	.1785
φ_p	70° 20′	71° 15′	70° 20′
$n = \tan \varphi_p$	2.798	2.946	2.798

Where $\rho_P \equiv$ minimum value of amplitude-ratio and $\varphi_p \equiv$ corresponding incidence, *i. e.*, the polarizing angle.

This verifies in a general way the turn in the dispersion curve in the green obtained by Professor Wood[1] and Mr. Pfund by means of thin-film interferometer measurements.

(*c*) In the following Table III. values of the phase difference are given for red light, (1) as observed, (2) as calculated on the assumption that selenium acts as a transparent medium, and (3) as calculated on the assumption of true metallic behavior of the selenium for red light. Judging from this table there is hardly any choice as to which set of formulæ to apply to the observations, for the observed values differ as much from one set of calculated values as from the other, and curiously enough in the same direction from both.

A comparison of the values of the refractive index secured by treating selenium as a vitreously reflecting substance with those secured by application of the equations of metallic reflection is presented in accompanying tabulation, which shows that in general the tangent of the polarizing angle gives a higher value of the refractive index than that calculated from the principal incidence and principal azimuth. The difference is greater in the case of fuchsin than for selenium.

A substance for which this transition from vitreous to metallic reflection as different wave-lengths are considered is even more

[1] Wood, Phil. Mag., June, 1902, p. 607; and also Proc. Phys. Soc. Lond., Vol. XVIII., pp. 607–622, 1902.

TABLE III.

Vitreous Selenium. $\lambda = 735\ \mu\mu$.

Angle of Incidence ϕ.	Phase Difference. Δ in π.		
	Observed.	Cal.[1]	Cal.[2]
30°	.0224	.0142	.0131
60	.0610	.1097	.0932
65	.0926	.1937	.1593
68	.2609	.3192	.2587
70	.4825	.4716	.3964
73	.7651	.7147	.6829
75	.8794	.8100	.8018
78	.9194	.8870	.8886
80	.9453	.9177	.9206

[1] Δ_π calculated on assumption that the selenium acts as a transparent medium for the red, $\phi_p = 70°\ 19'$, $\rho_p = .1095$; using the formula of Drude given on page 202.

[2] Δ_π calculated on assumption of true metallic behavior by means of Drude's formulæ (Optics, Eng. ed., p. 346; see p. 202, this paper), using $\bar{\varphi} = 70°\ 8'$, $\bar{\psi} = 6°\ 23'$, whence $k = .1988$, $n = 2.704$.

C: Comparison of Fuchsin and Selenium.

Fuchsin.

λ	455 $\mu\mu$	589 $\mu\mu$	634 $\mu\mu$
Principal incidence, $\bar{\varphi}$	48° 00′	67° 00′	64° 00′
Polarizing angle, ϕ_p	44 48	67 15	66 50
Refractive index, n (as cal. from $\bar{\varphi}$ and $\bar{\psi}$)	0.687	2.2307	2.0275
tan ϕ_p	1.010	2.385	2.337
Principal azimuth, $\bar{\psi}$	26° 50′	9° 40′	5° 15′
Absorption coeff't, k (as cal. from $\bar{\varphi}$ and $\bar{\psi}$).	.575	.293	.146
$k' = nk$	.3950	.6534	.2959

Selenium.

λ	432 $\mu\mu$	589 $\mu\mu$	735 $\mu\mu$
$\bar{\varphi}$	71° 34′	71° 06′	70° 08′
ϕ_p	70 20	71 15	70 20
n	2.811	2.8198	2.704
tan ϕ_p	2.798	2.946	2.798
$\bar{\psi}$	10° 20′	8° 55′	6° 23′
k	.3375	.2863	.1988
$k' = nk$	.9487	.8073	.5375

marked than for selenium or fuchsin, is nitroso-dimethylaniline. Professor Wood has shown that it is transparent to the ultra-violet while having a heavy absorption band in the blue region of the

spectrum and another in the extreme ultra-violet, being very transparent for yellow. He locates the center of the first band at 431 $\mu\mu$. Inasmuch as the refractive index cannot for this region be determined by direct prism measurements, I have studied its metallic reflection in a liquid form for $\lambda = 432\ \mu\mu$ in order from a knowledge of the principal incidence and principal azimuth to calculate the indices of absorption and refraction. Suffice it to point out here that for $\lambda = 432\ \mu\mu$ a phase-curve was secured very closely analogous to curve A in Fig. 1, while for yellow light, $\lambda = 589\ \mu\mu$, the nitrosodimethylaniline acted very nearly as a neutral liquid giving almost entirely plane polarized light for an incidence of 60° ± ; the region

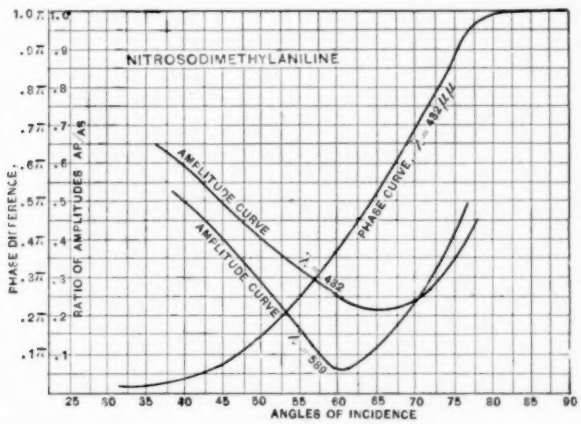

Fig. 7.

of the minimum on the amplitudes-curve being quite narrow. The results are shown in Fig. 7.

This comparison of the methods of interpreting the experimental results suggests that in all probability a result nearer the truth will be secured by applying the formulæ of metallic reflection to the observed values of azimuth and phase difference found for selenium for various incidences. These formulæ are given on page 201, and the result of applying them to the present observations to secure values of absorption coefficient and refractive index are given from here on.

(d) Effects of polish and scratches on phase difference (Δ) and azimuth (ψ).

The effect of polishing a mirror of vitreous selenium with chamois skin and rouge is shown in Table IV. There are also recorded for comparison the values of Δ and ψ for $\lambda = 432\ \mu\mu$ for a mirror which was badly scratched when the glass cover was knocked off in its preparation. The scratches were quite deep and slightly curved. They had a marked effect upon both the azimuth and the difference in phase, giving larger values of the azimuth and smaller values of the phase-difference than for the unpolished mirror. Those parts of the mirror which were not scratched had a much better mirror-surface than many of the other mirrors made. The

TABLE IV.

Effects of polish and of scratches on values of Δ and ψ.

Vitreous Selenium.

Kind of Mirror Vitreous Selenium.	$\lambda\mu\mu$	ϕ	ψ	δx	a[1]	Δ	Diff. in Δ
Polished.	589	68°	—	3.46	11.37	0.3027 π	.0315
Unpolished.	589	68	—	3.10	11.43	0.2712	
Polished.	735	65	13 05	1.87	14.35	0.1303	.0377
Unpolished.	735	65	10 24	1.32	14.26	0.0926	

Diff. in ψ, 241. $\lambda = 432\ \mu\mu$.

ϕ	ψ					Δ in π				
	Pol.	Diff.	Unpol.	Diff.	Scrat.	Pol.	Diff.	Unpol.	Diff.	Scrat.
65°	17° 13′	3° 37′	13° 36′	1° 05′	14° 42′	0.2598 π	.036 π	.2241 π	.002 π	.226 π
70	12 09	2 00	10 09	3 27	13 36	.4148	.020	.395		
75	13 13	1 13	12 00	3 45	15 45	.6555	.021	.634	.101	.533
80	19 49	2 06	17 43	3 23	21 06	.8700	.046	.824	.016	.808
Approx. Mean dif.	2°+		3°+				+.03π		−.04π	

effect on the phase is in agreement with the results of Drude[2] who showed that scratches on the mirror (made by rubbing it in a single direction with emery cloth No. 0000) in a direction parallel to the plane of incidence had little or no effect on the value of Δ, while scratches perpendicular to the plane of incidence

[1] The variations in a are due to the fact that the compensator was removed between these different observations and when replaced did not make the same angle with the light falling on it. But since δx as well as a is affected by the cosine of the angle which the face of compensator makes with the direction of the beam incident on it, the ratio $\delta x/a$, which alone enters the value of Δ, is unaffected.

[2] Drude, Wied. Ann., 39, 1890, pp. 481–554, especially in this connection pp. 497, 498.

gave Δ a little less than the normal value (*i. e.*, the value observed for as perfect mirrors as he could obtain). The difference, however, is small for the metals Drude investigated, amounting only to about $.003\lambda$–$.005\lambda$, while in the present case for vitreous selenium deeply scratched (some of the scratches were also wide), the decrease in phase-difference is $\pi.04$ or $.02\lambda$.

The effect of polishing with rouge and chamois was to increase Δ for all three wave-lengths, the change being slightly greater for the longer wave-lengths, *i. e.*, for those to which the unpolished selenium would be more transparent. This is in accord with previous work of Drude and others on the rôle played by the surface or transition layer, the condition of which is of capital importance in determining the metallic character of the reflection. The surface molecules are altered, perhaps in their manner of aggregation in such a way as to increase the apparent absorption coefficient, and naturally the effect of polishing a pseudo-metal like selenium by increasing the thickness of the transition layer, is more marked for those wave-lengths which before polishing were just on the border line between absorption and transparency.

The effect of polish on the values of the azimuth, ψ, as given by the observations recorded in Table IV., is, in general accord with Drude's results, which showed a larger value of ψ for the better mirror surfaces. The effect of the deep wide scratches in the selenium mirror was to increase ψ some 2° or 3°, and this is in general agreement with Drude's observation that a silver mirror rubbed in a direction normal to the plane of incidence with emery cloth No. 0000 gave ψ somewhat *greater* than for a perfectly good surface. On the other hand, he found that a silver mirror rubbed in all directions with emery cloth gave ψ some 16° *less* than the normal value. The scratches on the selenium mirror were, in general, normal to the plane of incidence, though being slightly curved they were in part somewhat parallel to the plane of incidence.

Scratches parallel to the plane of incidence would naturally be expected to increase the azimuth of the reflected light, since a scratch would reflect a vibration along its length but would have a destroying effect on a vibration at right angles to its length by absorbing it on account of multiple reflections from side to side.

Hence in the light reflected from a scratched mirror there would be a larger relative amount of light whose vibrations were parallel to the orientation of the scratches, and hence the resultant vibration would be turned toward the direction of the scratches and tan $\psi = A_p/A_s$ would be increased and so ψ, the azimuth, would be increased when the scratches are parallel to the plane of incidence.

The opposite would be expected for scratches normal to the plane of incidence.

The true azimuth for an unscratched surface could be secured then, from observations on a scratched mirror by taking the mean of two determinations of ψ made with the mirror in two positions with reference to a horizontal axis (in case the mirror is vertical) which differ by 90.

The values of Δ given in this paper are probably affected by scratches, etc., only within the error of observation, while the values of the azimuth may be a little too small on this account.

(c_{111}) *Metallic Selenium Mirrors.* — Table V. represents the observed values of ψ and Δ for different angles of incidence for a mirror of metallic selenium, made in the manner already described, for red, yellow and blue light. For red, Δ is considerably less for metallic selenium than for a vitreous mirror for all incidences greater than 65°, the same is true for yellow for $\varphi > 68°$ and for blue when $\varphi > 73°$. The effect on the blue is less marked than for yellow and red, and this is what we should expect, inasmuch as the blue is more heavily absorbed by the vitreous form than is either yellow or red, and so the change in Δ made by transforming the mirror from the vitreous to the metallic variety is greater for those wave-lengths for which the vitreous mirror was the more transparent or, rather, less absorbing than the metallic. In general the azimuth of the reflected light is greater for the metallic selenium than for the vitreous, except in the case of red light for $\varphi > 73°$.

The effect *in toto* of the transition from the vitreous to the metallic form is to make the selenium behave more like a true metal for all three wave-lengths investigated, the change being more marked for the longer wave-lengths. This is illustrated in the curves B, B, B in Fig. 2. For blue the change is very slight, indeed, and for red and yellow it is much less than was anticipated by reason of the

apparently large difference in the absorption coefficient for red; a film of amorphous selenium 0.1 mm. thick allowing a good deal of red light to pass, while a much thinner film of the metallic variety is entirely opaque.

TABLE V.

Polished Mirror of Metallic Selenium.

ϕ	Wave-Length: 735$\mu\mu$				589$\mu\mu$				432$\mu\mu$			
	Dist. bet. Bands: 16.80 Sc. Divs.				22.76.				28.60.			
	δx	Δ_π	ψ	A_p/A_s	δx	Δ_π	ψ	A_p/A_s	δx	Δ_π	ψ	A_p/A_s
		π				π				π		
65°	1.84	.129	12°15′	.217	2.38	.209	13°52′	.247	1.86	.221	17°06′	.307
68	3.43	.240	8 15	.145	3.72	.327	11 08	.197	2.43	.290	14 38	.261
70	4.54	.318	7 11	.126	4.10	.360	10 30	.185	3.27	.390	13 22	.237
73	8.62	.603	6 35	.115	6.64	.583	10 45	.190	4.22	.500	13 52	.247
75	11.12	.777	9 57	.175	7.48	.655	12 20	.219	5.42	.645	15 19	.274

REDUCTION OF OBSERVATIONS TO DETERMINE OPTICAL INDICES.

The values of the absorption coefficient (k and k') and the index of refraction (n) secured from the values of the principal incidence ($\bar{\varphi}$) and the principal azimuth ($\bar{\psi}$) observed by previous investigators together with those presented in this paper are given in the following Table VI. Columns 1, 1′ and 1″ have been calculated by the approximate formulæ; 2, 2′ and 2″ by the more rigorous formulæ; 3, 3′ and 3″ by the second form of the more rigorous formulæ. The results show how closely the last two agree, and also how approximate the first really is, giving considerably smaller values of n and larger values of k than the more rigorous formulæ, and of the two latter, the third set of formulæ gives consistently higher values for the absorption while giving identical values of the refractive index in comparison with the second set of formulæ; unless we apply, as I think we should, the formulæ given on page 203 to reduce $n_{\bar{\phi}}$ and $k_{\bar{\phi}}'$ to n_0 and k_0', in which case, as the columns 4 and 3 show, the third set of formulæ gives higher values of the refractive index also.

To secure values of the optical constants which shall agree well with those obtained by direct experiment, the observations of the phase difference and the azimuth must be confined to the immediate neighborhood of the principal incidence, and the second set of

Table VI.

Observed.				Calculated.				Calculated.			Calculated.			
Observer.	λ	$\bar{\Phi}$	$\bar{\psi}$	n				k			$k' = nk$			
				1	2	3	4	1′	2′	3′	1″	2″	3″	4″
Cornu.	280$\mu\mu$	68° 10′	10° 10′	2.172	2.350	2.343	2.333	.3706	.3146	.3186	.8049	.7393	.7465	.6841
" (old).	280	64 00	11 12	1.704	1.906	1.901		.4122	.3246	.3312	.7024	.6187	.6295	
Quincke.	431	70 10	15 30	2.235	2.381	2.381	2.339	.6009	.5275	.5297	1.3430	1.2560	1.2613	1.1757
* Edmunds.	432	71 34	10 20		2.811		2.794		.3375			.9487		.8984
Cornu.	439	71 30	5 00	2.791	2.947	2.944	2.943	.1763	.1576	.1586	.4921	.4644	.4669	.4403
" (old).	439	68 15	7 58	2.239	2.417	2.412		.2855	.2434	.2458	.6392	.5883	.5928	
Quincke.	486	71 00	13 14	2.458	2.604	2.603	2.5956	.4978	.4421	.4439	1.2236	1.1512	1.1554	1.0955
Glan.	508	71 28	7 31	2.731	2.885	2.884	2.863	.2686	.2400	.2439	.7335	.6924	.7033	
Quincke.	527	71 10	12 13	2.526	2.673	2.672	2.649	.4543	.4044	.4062	1.1476	1.0810	1.0855	1.0202
Quincke.	589	71 23	11 32	2.588	2.735	2.733	2.714	.4358	.3801	.3820	1.1279	1.0400	1.0440	.9832
* Edmunds.	589	71 19	8 54		2.8198		2.8076		.2863			.8073		.7637
Glan.	631	69 49	4 8	2.527	2.697	2.692	2.694	.1453	.1269	.1279	.3672	.3422	.3443	.3212
Quincke.	656	71 26	8 24	2.702	2.854	2.851	2.843	.3019	.2697	.2711	.8157	.7697	.7729	.7287
Cornu.	721	69 35	0 30	2.517	2.692	2.686	2.692	.0174	.0152	.0153	.0438	.0409	.0412	.0387
" (old).	721	69 00	7 00	2.360	2.534	2.529		.2493	.2151	.2171	.5883	.5451	.5490	
* Edmunds.	735	70 08	6 23		2.704		2.698		.1988			.5375		.5052

VITREOUS SELENIUM.

Indices of refraction (n_0), absorption coefficients (k_0'), and reflecting power (R) calculated from observed values of phase difference (Δ) and azimuth (ψ) in neighborhood of principal incidence ($\bar{\phi}$).

	$\lambda = 432\ \mu\mu$				589 $\mu\mu$			735 $\mu\mu$		
ϕ	69° 40′	70° 40′	71° 40′	72° 40′	70°	71°	72°	69° 10′	70° 10′	71° 10′
ψ	14° 47′	14° 25′	14° 46′	15° 47′	13° 28′	13° 35′	13° 19′	6° 41′	7° 01′	7° 55′
Δ	.4592 π	.5371 π	.5849 π	.6244 π	.4747 π	.5444 π	.5927 π	.4300 π	.5333 π	.61695 π
log S	.39991	.40540	.40031	.39776	.39625	.41114	.41018	.36850	.40545	.40073
log tan Q	$\bar{1}$.75377	$\bar{1}$.73781	$\bar{1}$.73759	$\bar{1}$.75438	$\bar{1}$.70455	$\bar{1}$.70603	$\bar{1}$.68158	$\bar{1}$.36529	$\bar{1}$.39546	1.42272
n_i	2.3366	2.3852	2.3632	2.3314	2.3796	2.4522	2.4762	2.4579	2.6372	2.6044
k_i	.49332	.47636	.47376	.49080	.4391	.4441	.4189	.1975	.2168	.2297
$n_i k_i = k_i'$	1.1527	1.1365	1.1197	1.1445	1.045	1.089	1.0372	.4855	.5716	.5982
n_0	2.2985	2.3495	2.3271	2.2921	2.3487	2.4214	2.4483	2.4511	2.6296	2.5958
$k_0'(=n_0k_0)$	1.0733	1.0597	1.0413	1.0621	.9727	1.0176	.9686	.4503	.5356	.5391
	Means (1) $n_0 = 2.3168$			$k_0' = 1.059$	Means (1) $n_0 = 2.406$		$k_0' = .9863$	Means (1) $n_0 = 2.559$		$k_0' = .5150$
By interpolation using LaGrange's formulæ,										
	$\bar{\phi} = 69° 46'$,			$\bar{\psi} = 14° 43'$	$\bar{\phi} = 70° 22'$,		$\bar{\psi} = 13°31'$	$\bar{\phi} = 69° 50'$,		$\bar{\psi} = 6° 54'$
Whence by Walter's formulæ:										
	(2) $n_0 = 2.3674$			$k_0' = 1.171$	(2) $n_0 = 2.500$		$k_0' = 1.129$	(2) $n_0 = 2.645$		$k_0' = .5716$
Reflecting power,	$R = 25.50\%$ from (2)				$R = 26.06\%$ from (2)			$R = 22.28\%$ from (2)		
	23.53 " (1)				24.09 " (1)			20.78 " (1)		

formulæ applied. The observations already cited having given for the latter the following values :

$$\lambda = 432\,\mu\mu \qquad 589\,\mu\mu \qquad 735\,\mu\mu,$$
$$\bar{\varphi} = 71^\circ\ 34' \qquad 71^\circ\ 19' \qquad 10^\circ\ 08',$$

observations of phase difference (Δ) and azimuth (ψ) were made on the same mirror (now five months old) as shown by the following summary.

The index of refraction is found to increase with increasing wave-length for an old mirror, and this agrees with Quincke's observations and tends to confirm the view that his mirrors of selenium were not used till some time after making.

Effect of Time on the Principal Incidence and Principal Azimuth. — In order to show the effect of time on the values of the principal incidence ($\bar{\varphi}$) and principal azimuth ($\bar{\psi}$), I shall apply the same method to both mine and Cornu's observations of these constants of elliptical polarization for a fresh and an old mirror. The results are presented in Table VII. Walter's formulæ were used in calculating the index of refraction n_0, the absorption coefficient k_0', and the reflecting power R, from the principal incidence and azimuth.

Lapse of time decreases the principal incidence some 18′ to 1° 48′ after five months interval, the decrease being greater for the shorter wave-lengths. An interval of seven years gave a much larger decrease of from 0° 35′ to 4° 10′ in $\bar{\varphi}$, the greatest change being for the ultra-violet. Interpreted in terms of phase-difference, this means that the surface has changed in such a way as to increase the phase-difference (Δ) for any given incidence and to make the value $\Delta = \pi/2$ occur at a smaller incidence.

The effect of time is to increase the principal azimuth by several degrees in general ; no simple variation of this change with wave-length appears and the change after five months is only 31′ for the red. Since, roughly speaking, the absorption is directly proportional to the tan $\bar{\psi}$, the observations show an increase in absorption for all wave-lengths as time elapses.

The effect of these variations in the values of the principal incidence and principal azimuth with time is to decrease the index of refraction and to increase the absorption coefficient, as calculated

TABLE VII.

Effect of Time on Vitreous Selenium.

Observer: Cornu. Time Interval: 7 Years.

Mirror.	$\lambda = 280\mu\mu$					$\lambda = 439\mu\mu$					$\lambda = 721\mu\mu$				
	$\bar{\Phi}$	$\bar{\Psi}$	n_0	k_0	R	$\bar{\Phi}$	$\bar{\Psi}$	n_0	k_0	R	$\bar{\Phi}$	$\bar{\Psi}$	n_0	k_0	R
New.	68° 10′	10° 10′	2.343	.746	20.08 %	71° 30′	5° 00′	2.944	.467	25.34 %	69° 35′	0° 30′	2.686	.041	21.01 %
Old.	64 00	11 12	1.901	.630	13.71	68 15	7 58	2.412	.593	19.55	69 00	7 00	2.529	.549	19.91
Diffs.	−4 10	+1 02	−0.442	−.116	−6.37	−3 15	+2 58	−0.532	+.126	−5.79	−0 35	+6 30	−.157	+.508	−1.10

Observer: Edmunds. Time Interval: 5 Months.

Mirror.	$\lambda = 432\mu\mu$					$\lambda = 589\mu\mu$					$\lambda = 735\mu\mu$				
	$\bar{\Phi}$	$\bar{\Psi}$	n_0	k_0	R	$\bar{\Phi}$	$\bar{\Psi}$	n_0	k_0	R	$\bar{\Phi}$	$\bar{\Psi}$	n_0	k_0	R
New.	71° 34′	10° 20′	2.811	.949	27.10 %	71° 06′	8° 55′	2.820	.807	26.00 %	70° 08′	6° 23′	2.704	.538	22.79 %
Old.	69 46	14 43	2.367	1.171	25.50	70 22	13 31	2.500	1.129	26.06	69 50	6 54	2.645	.572	22.28
Diffs.	−1 48	+4 17	−.444	+.232	−1.60	−0 44	+4 36	−.320	+.322	+0.06	−0 18	+0 31	−.069	+.034	+0.51

from these constants of metallic reflection. Both Cornu's observations and mine show the change in the refractive index to be considerable and to be nearly as great after five months as at the end of seven years.

My observations show the absorption coefficient to increase considerably, though less in the red than for yellow and blue; while Cornu's values give a decrease for the ultra-violet and a large increase in the blue and red.

The dispersion and absorption curves obtained by plotting values given in columns 1, 2 and 3 of Table VI. are shown in the accompanying diagram (Fig. 8) together with those directly observed by Sirks and by Wood[1] and Pfund.

The general discrepancy between the results of the several observers is very marked, especially for the absorption coefficient. The difference between the observed values of the principal incidence and principal azimuth of various observers are shown graphically in Fig. 9 and are probably to be accounted for in great part by the influence of scratches, polish, age, etc., as already demonstrated in this paper. The values of the absorption coefficient calculated from these metallic reflection experiments differ as much among themselves as they do from the directly observed values.

COMPARISON OF RESULTS.

In the metallic reflection experiments we are really studying surface conditions only, whereas in the experiments in which direct observations are made for absorption and refraction we are studying phenomena, having their seat inside the medium. In the metallic reflection experiments no doubt the surface conditions have varied widely and in an unknown manner. In the direct measures of absorption, the selenium was in a form entirely different from that employed in the metallic reflection experiments. Wood and Pfund used their films of amorphous selenium secured by cathode deposit, and close examination of these films shows that they are not perfectly continuous but are rather of a spongy structure which accounts in part for the much lower values of absorption secured by these observers, although their neglect to take into account the loss by reflection at the first surface would certainly tend to balance this. Their direct measures of the refractive index, n, were in the red and yellow region of the spectrum made with small-angle prisms of melted vitreous selenium, and as might be expected, there is a

[1] R. W. Wood, Phil. Mag., June, 1902, p. 607.

closer agreement in that region with the values of n calculated from the metallic reflection experiments. In the green-violet re-

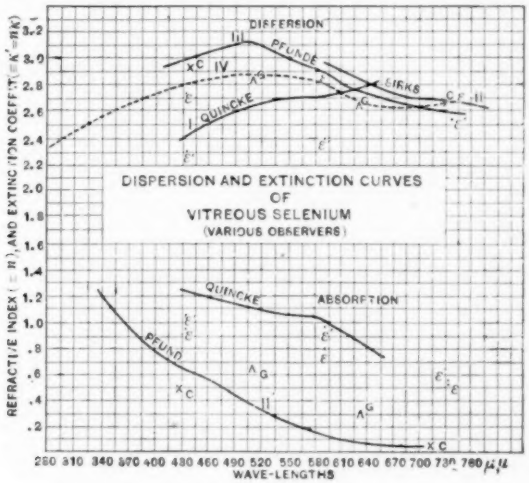

Fig. 8.

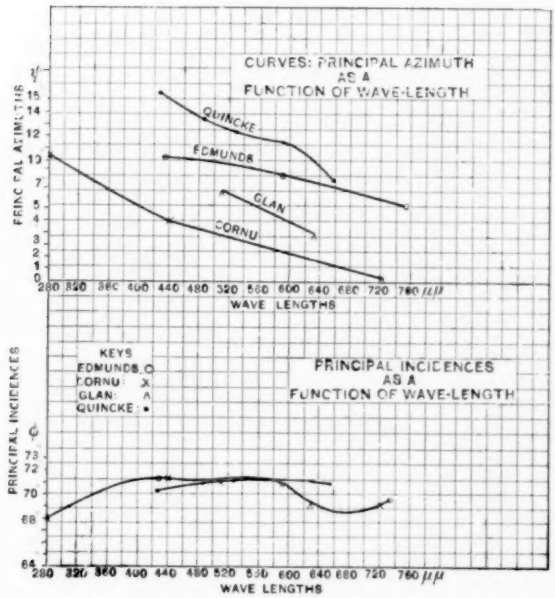

Fig. 9.

gion the direct measures of n were made by interferometer measurements on very thin films of amorphous selenium, and there is some doubt as to the validity of these results.

TABLE VIII.

Index of refraction and absorption coefficient of vitreous selenium.

Column:	1	2	3	4	5	6
Wave-Length	n Direct.	Cal. n by Reflec.	kn Direct.	kn Cal. Metal. Reflec.	kn Cal. Reflec.	kn Cal. Reflec.
214$\mu\mu$						
280		2.333		.6841		
334						.9767
395						.9614
400	2.93		.785			
410	2.957					1.0275
415	2.910		.736			
425	2.98		.677			
430						1.1437
431		2.381		1.2613		
432	3.00	2.811		.9487	1.1776	
439		2.947		.4669		
442	3.025		.598		.8205	
454						
466	3.065		.571			
475	3.075					
486		2.604		1.1554		
490	3.120		.478		.4544	
503	3.130					
508		2.885		.7033		
510						1.1354
515			.358			
520	3.085					
524						
527		2.673		1.0855	.7232	
550	3.020		.254			1.0020
580	2.95	2.735		1.0440		
589	2.925	2.8076	.155	0.7637	.7813	.8827
605	2.90					.8415
620					1.1916	1.0728
631		2.697		.3443		
640	2.765		.0879			
656		2.854		.7729	1.1046	
660						
710	2.655		.0456			
721		2.692		.0412		
735		2.704				
748				.5376	.3478	
760	2.61		.0234			

Observers: Cols. 1 and 3, Pfund and Wood.
Cols. 2 and 4, Cornu, Quincke, Glan, Edmunds.
Col. 5, Edmunds.
Col. 6, Nutting (cal. by Edmunds).

From the data at hand, it is extremely uncertain just what form to assign to the dispersion curve of selenium. Quincke's observations, though covering a good portion of the visible spectrum, give, no matter from what point of view we consider them, results which are contrary to those of all other observers. His values of refractive index increase with increase of wave-length, and his values of the absorption coefficient are very much higher than any other determinations. I am at a loss to explain his results except that they may possibly be due, as the high absorption would suggest, to the fact that his mirrors were made some time before he actually observed with them, and so had a chance to pass over into the gray crystalline or metallic form, the apparent absorption of which he further increased by polishing just before observing.

The present work on the metallic reflection of selenium was undertaken to secure merely check values of refractive index and absorption coefficient for two or three wave-lengths in the visible spectrum. It would seem, however, desirable to make a more extended study so as to be able actually to plot a full dispersion curve from a single set of experiments.

Neglecting Quincke's observations, the values of refractive index secured by other determinations of the principal incidence and principal azimuth lie on a smooth dispersion curve which agrees in general "march" with that observed directly. The discrepancy in the values of the absorption coefficient is, however, very great, and further experiments are required to determine the extinction curve of selenium. All that the present observations enable us to say definitely is that the absorption increases from the red toward the blue end of the spectrum. Some experiments attempting to secure "Reststrahlen" in the ultra-violet by multiple reflections from selenium surfaces showed no great increase in reflecting power in that region; if anything, there was a falling off.

Lack of time and a desire to measure the reflecting power of selenium directly by photometric methods have as yet prevented the determination of more accurate dispersion and extinction curves.

The values secured from the present experiments with the fresh mirror for the wave-length $589\mu\mu$ will be used as a basis in calculating the reflecting power for different angles of incidence for com-

parison with the directly observed values of R. And in determining the absorption curve from the experimentally determined reflection curve for normal incidence, the dispersion curve resulting from all the experiments on metallic reflection, excepting those of Quincke, will be adopted. In this way all the quantities used and compared will result from experiments in which the chief rôle is played by the surface, and so there will be a common ground for the comparison.

Summary of Experiments on Metallic Reflection.

The present experimental work on the metallic reflection of selenium has shown that the elliptical polarization of plane-polarized light by reflection from vitreous selenium is such as is characteristic of a transparent substance for wave-lengths in the red, and that the reflected light assumes more and more the characteristics of light reflected from true metal surfaces as the wave-length of the incident light is decreased, till in the blue selenium acts quite like a metal. In this respect selenium is somewhat analogous to fuchsin.

The variation of the principal incidence and the principal azimuth with changing wave-length was noted. The principal azimuth increases as the wave-length is decreased, while the principal incidence passes through a minimum at about 680$\mu\mu$ and then remains fairly constant as the wave-length is decreased.

The effect of transforming the vitreous selenium into the gray crystalline or metallic form is to increase the absorption for all wave-lengths in the visible spectrum and to render the condition of the elliptically polarized reflected light more like that from a true metal, the change being more marked for yellow light than for blue or red.

Scratches on the surface of the mirror increase the value of the azimuth and decrease the difference in phase of the two components of the reflected light, for any given angle of incidence.

Polishing the mirror with rouge and chamois increases the phase-difference, the change being greater for the wave-lengths to which the unpolished selenium is more transparent. Polishing increases the value of the azimuth for all wave-lengths.

Lapse of time makes the vitreous selenium behave more like a metal, decreasing the principal incidence and increasing the princi-

pal azimuth. This is doubtless due to the formation of a surface layer in which the conditions are very different from those of interior portions of the selenium.

The analytical interpretation of the elliptical polarization phenomena by means of formulæ developed chiefly by Drude showed discordant values of the refractive index and absorption coefficient when the observations of different observers are compared. In general, however, the dispersion curve rises as the wave-length is decreased and has a turning point in the green at about 505$\mu\mu$. The absorption rises gradually as the wave-length is decreased, and all the experiments as well as the application of the dispersion formulæ seem to indicate a number of overlapping absorption bands in the blue-violet region of the spectrum.

The experiments indicate that the closer and more extended study of the absorption and dispersion of selenium will doubtless present a very interesting case to which as a test the various formulæ of metallic reflection and of the dispersion of metals may be applied.

JOHNS HOPKINS UNIVERSITY,
PHYSICAL LABORATORY, May, 1903.

RESONANCE IN AËRIAL SYSTEMS.

BY A. H. TAYLOR.

THE object of this paper is to outline a simple method of attaining resonance in systems subject to electrical oscillations of high frequency, and to point out several unique lecture-room demonstrations of phenomena peculiar to such systems.

As has been pointed out by Wien (Ann. der Phys., page 686, No. 7, 1902) and others, sharp resonance effects occur only when the damping is small compared to the energy supply. Such an arrangement as is shown in Fig. 1, p. 233, is one to give little damping in the non-radiating, low-resistance circuit, consisting of the condenser, *C*, a spark gap, and the low self-induction, *P*, of the primary of a Tesla coil.

The aërial *A* is coupled magnetically to the system by being connected to the secondary of the Tesla coil *S*, one end of which is earthed. Such an arrangement has been used by Marconi, Braun, Fleming and many others.

The damping of the oscillations in the aërial is very great, but the energy supply from the primary circuit is also great, so that the total damping is small.

The radiation from the aërial depends on the current flowing in it, and this will be a maximum when the natural period of the secondary of the Tesla with its aërial system is the same as the primary period. The secondary period depends on the self-induction of the secondary, of the aërial itself, and upon the capacity of the aërial system relative to earth.

Any measuring device inserted above the secondary *S* is liable to change the relative capacity and hence the period. Even a micrometer spark-gap to measure the potential of the wire to earth will change the capacity of the aërial very materially. The simplest way is to insert a short piece of platinum wire in the earth connection *E*. Being on the earthed side of the system, a Wheatstone's

bridge connected to this wire will not change the capacity of the system. The platinum wire may be about 2 cm. of No. 36, and forms one arm of the bridge. The bridge is balanced, and the exciter, a 6″ induction coil used as a step-up transformer, with 10 amperes alternating current at 60 cycles, is turned on. The flow of current in and out of the aërial, and hence in and out of the earth connection, is in nearly all cases large enough to heat the platinum wire and effect a change in resistance of sufficient magnitude to be easily noted with a galvanometer of moderate sensibility, say 10^{-7}.

Then coils of self-inductance are added to the aërial, just above the secondary *S*.

If the galvanometer does not show an increased deflection, hence an increased resistance of the platinum wire, and more current in the aërial, then the capacity of the aërial combined with the self-induction of the secondary *S*, gives a period already slower than that of the primary. In this case a smaller aërial system will have to be used or else the primary period must be lengthened by added capacity or self-inductance in the primary circuit. When once the primary period is slower than that of the secondary and ëarial, added self-induction to the aërial will at first produce a slow increase in the galvanometer deflection, then as syntony is nearly reached, a very rapid increase, attaining a maximum beyond which further addition of self-induction produces a decrease in the deflection. Self-induction is best used in the form of two concentric helices, connected in series so that the mutual induction between the two opposes the self-induction in each. By sliding the inner helix in and out the self-inductance can be smoothly varied, and the resonance point very quickly determined.

The deflections will indicate that the maximum current is often as much as one hundred times as large as the current when the aërial is far out of resonance.

Having the radiating aërial resonating the primary circuit, it is now necessary to have a second primary circuit in tune with the first.

The second Tesla coil should be made with the secondary self-induction as near as possible like that of the first. The primaries should be approximately alike. The second outfit is then tuned to

the aërial by adjusting the small variable self-induction p in the primary circuit.

The method is the same as when tuning the aërial to the first primary.

Now we have two oscillators in tune with one aërial, hence, with each other. If they are to stay in tune they must be rigidly mounted, and used in the same relative positions.

The secondary primary may now be removed to the receiving station and a new aërial tuned to it, in which case the two primaries are in tune with each other and with the two aërials. This four-fold resonance is absolutely necessary for the best results.

In case the aërial is coupled, as in Fig. 2, the method is the same, the platinum wire being inserted on the ground side as before. In this case the aërial, with its capacity to earth and inductance, including P, must resonate the impulses in E.M.F. over the induction P considered as part of the primary oscillating circuit.

The case is the same as when a Tesla coil is used, only here we have an auto-transformer coupling of ratio 1 to 1.

The resonance of aërial systems may be very elegantly demonstrated in the lecture room by placing a small lamp, say 1 candle power at 4 volts, either directly in series with the receiving aërial, or across the spark knobs of the primary circuit of the receiver. If an ordinary direct current make and break is used on the coil, the radiated energy attains to high instantaneous values, such as are peculiarly suited to the operation of a coherer or a magnetic detector, but the average energy is far lower than when the alternating current is used, without an interrupter. Hence, to transmit across a lecture room, say a distance of twenty feet, enough energy to light the lamp in the receiving circuit it is necessary to operate the induction coil with alternating current. In this instance a 6″ spark coil carried 15 amperes at 60 cycles and 40 volts. The power factor was at first very low, but as the transmitting aërial came into resonance, the power factor ran as high as 0.5. A spark gap of 2–4 mm. in the primary circuit must, if operated continuously, be cooled by an air-draft to prevent arcing.

This is not ordinarily necessay, as the coil need be run only for short intervals.

In the lecture room the sending aërial may consist of a strip of wire netting about ten feet by two feet, suspended as near the ceiling as possible, and coupled either by the auto-transformer method (Fig. 2) or by the Tesla transformer (Fig. 1) to the sender. The variable self-inductance should be in series with the aërial.

The receiving aërial is similar, and may be used with or without the condenser circuit similar to the primary sender circuit. If the condenser circuit is used, the little lamp is placed across the spark-gap, or rather where the spark-gap would be if the receiver were used as a transmitter. If the circuits have been tuned carefully, the lamp in the receiving circuit will burn brightly and steadily as long as the transmitter operates. If either aërial is thrown out of resonance by varying slightly the self-inductance in series with them, the lamp goes out entirely.

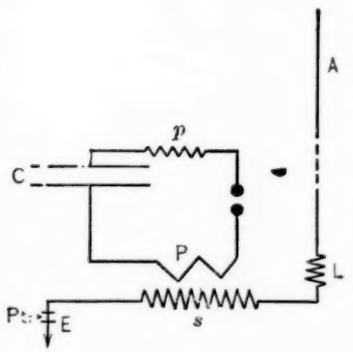

Fig. 1.

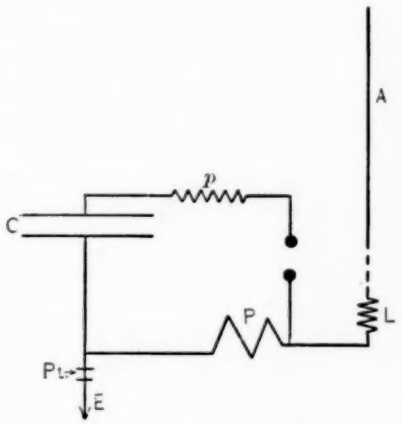

Fig. 2.

If desired, a hot wire voltmeter or ammeter can be used at the spark-gap of the receiving circuit in place of the lamp. Under fair conditions, I have gotten an indication of over an ampere at the receiver, distant 20 feet from the transmitter. This drops to a few tenths of an ampere when either aërial is thrown a little out of resonance. The Tesla coil coupling is best suited for the use of an ammeter, since in the receiver it acts as a step-down transformer. The real current is doubtless somewhat less than that indicated, due to the fact that the ammeter wire has higher resistance with such high frequencies (of the order of 10^6) than it has for ordinary

frequencies. The small lamp is better for lecture room demonstrations.

The screening effect of conducting obstacles may be illustrated by interposing a wire netting screen between the two aërials, causing a dimming of the lamps in the receiving circuit. Either aërial may be thrown perceptibly out of tune by a person standing near it, or by holding near it a conductor connected to earth, thereby changing its capacity. A conductor connected to earth may be suspended near the transmitting aerial, and the aërial tuned accordingly to its new capacity by adjusting the variable inductance in series with it. The radiation under these conditions is, however,

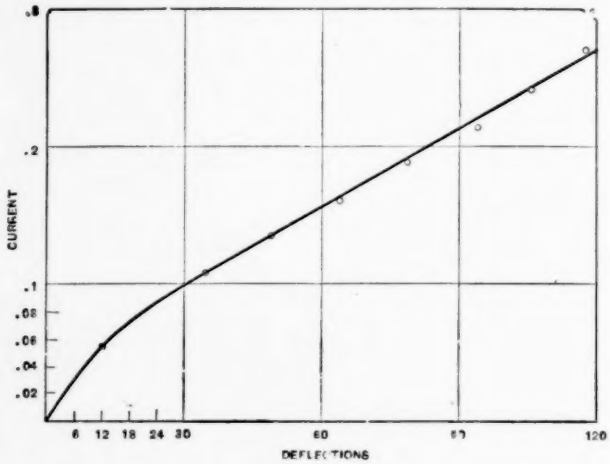

Fig. 3.

less than before, as shown by the dimming of the lamp in the receiving circuit. This illustrates the disadvantage of having earthed conductors in the neighborhood of an aërial wire.

In order to plot a resonance curve for either of the aërials it is necessary to know the magnitudes of the currents in them. To this end the platinum wire used on the ground side of the receiving aerial was replaced by the 4-volt, 1 candle-power, incandescent, and balanced in one arm of a Wheatstone bridge. Then alternating currents at 60 cycles were sent through the lamp while still in the bridge arm. This heated the lamp, lowering its resistance (being a carbon filament) and caused the galvanometer to deflect. The heating current was measured by sending it through an Olivetti hot wire voltmeter of known resistance.

Curve I. shows the relation between heating currents and deflections of the bridge galvanometer.

The lamp, still in the bridge arm, was now put back directly in the receiving aërial, and the galvanometer deflections observed as the self-induction in series with the aërial was varied, while there was a steady radiation of energy from the transmitter.

By reference to Fig. 3, the galvanometer deflections were translated into equivalent currents and Curve II. plotted. The natural period of the aërial would be proportional to the square roots of the localized self-induction, the capacity, also principally localized, being constant.

Hence, in Fig. 4, I have plotted current in aërial and square roots of self-induction.

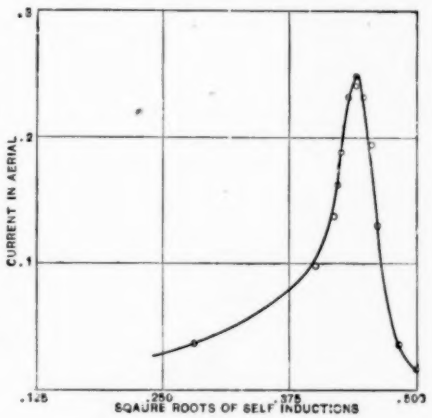

Fig. 4.

The self-induction was varied from 0.16 to 0.25 millihenrys. These self-inductions are certainly not correct for such high frequency, as they were obtained by calibration with a standard at low frequency. However, the difference is not large enough to affect the form of the resonance curve. For Curve II. the auto-transformer coupling of the transmitter was used, and the receiver consisted of four electric light cords, suspended from the ceiling, and all in series with a variable and previously calibrated self-inductance. The receiving aërial was 10 feet high and about 20 feet from the transmitter. The resonance point is fairly sharp and corresponds to a self-induction of 0.195 millihenrys. The capacity of the aërial was measured by charging it to 50 volts and discharging it into a standard condenser one hundred times in succession. The standard condenser showed no appreciable leak in 15 minutes and three determinations of the receiving aërial capacity agreed within 1 per cent. of $1.85 - 10^{-10}$ farads. This gives a period of 1.19×10^{-6}. If the self-induction were calculated or measured for high frequency this period would be slightly decreased.

Curve V. shows the increased sharpness of the resonance where the receiver as well as the transmitter is provided with an auto-coupling, and the little lamp placed as indicated in Fig. 5.

The receiver condenser circuit has the same period as the transmitter. Hence when the receiving aërial is out of tune with the transmitter, it is also out of tune with its own condenser circuit. and the resonance is very marked.

It will be noted that the currents are not nearly so large as in the aërial itself. The sharper resonance is attained at this expense, adapting this system, as Wien (*loc. cit.*) remarks, to short distances in territory in which are many other receivers, thus necessitating sharp tuning, and not requiring much energy.

Fig. 6 is obtained with a simple aërial for receiver, and the Tesla coupling for the transmitter. The resonance is not very sharp, but

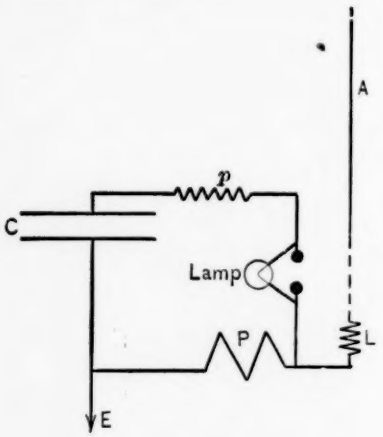

Fig. 5.

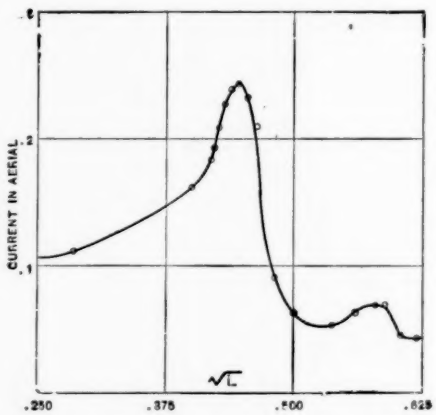

Fig. 6.

occurs for the same frequency as for Curves II. and V. Wien (*loc. cit.*) has shown that a magnetically coupled system radiates two trains of waves of different frequencies, and, in general, different damping coefficients. The lower resonance point at $\sqrt{L} = 0.575$ is for a period of 1.63×10^{-6} and doubtless corresponds to the second and less predominant wave-train. The damping is evidently much greater for this wave-length.

Fig. 7 shows the resonance effect when both sender and receiver are provided with the Tesla coupling, and the lamp is put across

the spark-gap as in Fig. 3. The Tesla coil in the receiver acts as a step-down transformer, and hence the current is greater than in the aërial itself. The effective self-induction of the secondary of the Tesla coil of the receiver was 0.023 millihenrys, so that in order to get the range of self-inductance within the limits of the calibrated variable self-inductance, it was necessary to use an aërial of somewhat smaller capacity. This shifts the resonance point to $L = 0.237$, and also decreases the current in the receiver. If the same aërial had been used as for Curve III., the currents would have been even greater for Curve IV. It will be noted that the resonance, although much sharper than for a single aërial receiver, is not nearly as sharp as for the case represented by Curve V.,

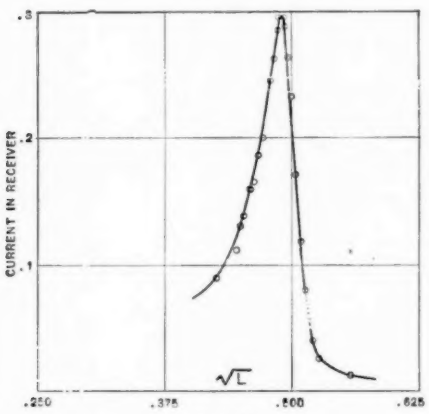

Fig .7.

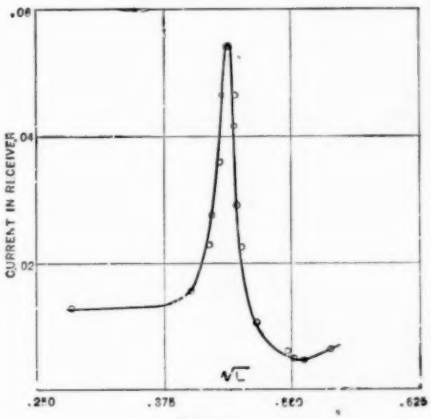

Fig. 8.

while the energy received is very much greater. This also agrees with Wien's conclusion that the Tesla coupling is suitable for long distance work and where sharp tuning is not necessary. The curves are all steeper on the side towards greater self-induction, as is characteristic of electrical resonance curves in general.

If the secondary of the Tesla coil is easily removable, it may be taken out and inserted in series with the aërial, and the latter joined to the primary by the auto-coupling. The aërial system will then still be very nearly in tune, a very slight decrease in self-inductance of the aërial being advisable, to allow for the mutual induction between primary and secondary, which is now absent. In this way one can pass from one coupling to the other without stopping to tune the

transmitter each time. If the primary has no "ballasting" self-induction (as at p, Fig. 1) the transition is not quite so easy, since the lack of mutual induction now lengthens the primary period appreciably.

If the receiving circuit is far distant from the sender, one of them can be tuned to a small portable system consisting of a condenser with a variable inductance in series. The other system can then be also tuned to the portable system and both systems checked up occasionally to see if they are in tune.

PHYSICAL LABORATORY, UNIVERSITY OF WINCONSIN,
Madison, January 6, 1904.

ON THE CONDUCTIVITY PRODUCED IN RAREFIED GASES BY AN INCANDESCENT KATHODE.[1]

By Ernest Merritt and Oscar M. Stewart.

THE increased electrical conductivity of rarefied gases produced by the employment of an incandescent kathode seems to have been noticed almost simultaneously by Hittorf[2] and by Edison. A result of this increased conducting power is strikingly shown in the case of the "Edison effect"[3] where the relatively small fall of potential in a glow lamp is sufficient to produce a current of several milliamperes through the gas separating the two ends of the filament, or between the filament and a third terminal which is positive with reference to it. The destruction of the filament can often be traced to this leakage current through the gas. Elster and Geitel have found that insulated conductors acquire a charge when placed near an incandescent solid in a partial vacuum, and have made an extended study of this effect. The explanation offered by J. J. Thomson[4] refers both of these phenomena to a common cause, namely the emission by the incandescent solid of negatively charged particles, which may either impart a static charge to a neighboring insulated body, or may act as the carriers of a current between the incandescent kathode and any suitable anode. Determinations of the ratio of charge to mass in the case of these particles indicate that they are similar to those which constitute ordinary kathode rays.[5]

There are many points of similarity between the behavior of an incandescent kathode and that of a kathode of suitable material, such as zinc, when illuminated by ultra-violet light. At sufficiently

[1] A preliminary account of the work here described was presented to the Physical Society, Feb. 22, 1902. (See Science, Vol. 15, p. 425.)

[2] Hittorf, Wied. Ann., 21, p. 119, 1884.

[3] Houston, Trans. Am. Inst. E. E., Oct., 1884, Vol. I.; Fleming, Phil. Mag., July, 1896; Preece, Proc. Roy. Soc, 38, p. 219; Elster & Geitel, Wied. Ann., 37, p. 315; Howell, Trans. Am. Inst. E. E., 14, p. 27, 1897.

[4] J. J. Thomson, Conduction of Electricity through Gases, Chap. VIII.

[5] J. J. Thomson, Phil. Mag., Vol. 48, p. 547, 1899.

high vacua true kathode rays appear to be developed in both cases. In one case these are due to the action of heat; in the other case to the action of short light waves. The experiments here described were suggested by this similarity between the two classes of phenomena.

In the case of the photo-electric discharge the current may be looked upon as consisting of two parts: (1) The current carried by the kathode rays produced by illumination; (2) the current carried by the ions that are developed by these rays in the surrounding air. That ionization is really produced by the slowly moving kathode rays of the photo-electric discharge has been shown by direct experiment.[1] We are not aware that there is any direct proof that the kathode rays from an incandescent solid possess the same ionizing power. But it seems highly probable that they do. If so, the current in the case of the Edison effect must also consist of two parts, one carried by the kathode rays themselves and the other by the ions produced by them. At high vacua we should therefore expect the phenomena to appear in their simplest form, since the conducting power of the slight remaining traces of gas would in all probability be inappreciable. For this reason our first experiments were made at high vacua, and experiments were afterwards made at pressures in the neighborhood of 1 mm., where the conducting power of the air proved to be an important factor. In the main the results obtained were such as would be anticipated from the above-mentioned analogy. But at high pressures a phenomenon was observed whose analogue in the case of the photo-electric discharge has not heretofore been detected.

APPARATUS.

The apparatus was arranged as shown in Fig. 1, which shows the lamp at half its actual size. The filament[2] was of low resistance, and in our experiments took a current of from 1.2 amp. to 1.7 amp. with a p. d. between its terminals ranging from 6 volts to 8 volts. The filament was surrounded by the brass cylinder *CC* which served as anode. A galvanometer served to measure the

[1] Merritt and Stewart, PHYSICAL REVIEW, Vol. XI., p. 230, 1900.

[2] A number of filaments specially mounted for this purpose were kindly furnished by the Edison lamp works.

current between *CC* and the filament. A battery *D* of adjustable e.m.f. in series with the galvanometer gave the desired potential difference between *F* and *CC*. The galvanometer current was in all cases so small that the resistance of the galvanometer and connections produced no appreciable fall of potential, so that the potential difference between *CC* and *F* was the same as the e.m.f. of the battery.

In most cases the circuit containing the galvanometer and the battery was connected to the negative terminal of the filament. The positive pole of the battery was connected to the cylinder *CC*, thus making it the anode. When the e.m.f. of the battery was zero, the cylinder was at the same potential as the negative terminal of the filament. In this case there would be no galvanometer current, since the discharge takes place only from that part of the fila-

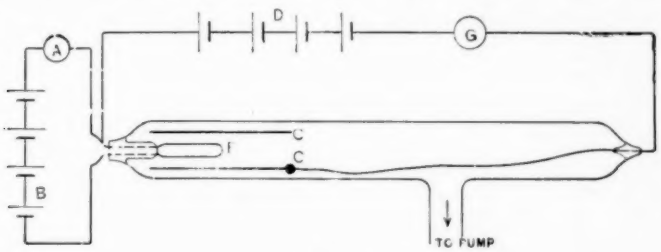

Fig. 1.

ment that is at a *lower* potential than the other conductor. But when the e.m.f. had any value greater than zero the galvanometer was deflected.

Upon reversing the battery *D* so as to make the filament the anode, the galvanometer showed no current unless the filament was heated to high incandescence; and even then the current was extremely small. This reversed current, already noticed by Fleming, has not been investigated in the work here described.

The magnitude of the current between *CC* and *F*, for a given e.m.f. and with all other conditions constant, was found to depend very greatly upon the degree of incandescence of the filament. This is well shown in Fig. 2. The upper curve is for an e.m.f. of 1.4 volts, or one Leclanche cell in the battery *D*, while the lower curve corresponds to an e.m.f. of 11 volts, or 8 cells. The vertical scale in the latter case is $\frac{1}{10}$ that of the former.

Since the galvanometer current was so greatly altered by changes in the lamp current, great care had to be taken to keep the latter constant. A storage battery was used as a source of e.m.f. But in determining the curves described below even the small fluctuations in the current from this battery were sufficient to introduce errors. An ammeter was therefore kept in the circuit and the current was adjusted, when this was necessary, before each reading.

A McLeod gauge was kept in connection with the pump through, out the work. Very little reliance can be placed on the indications of this gauge, so far as absolute values of pressure are concerned,

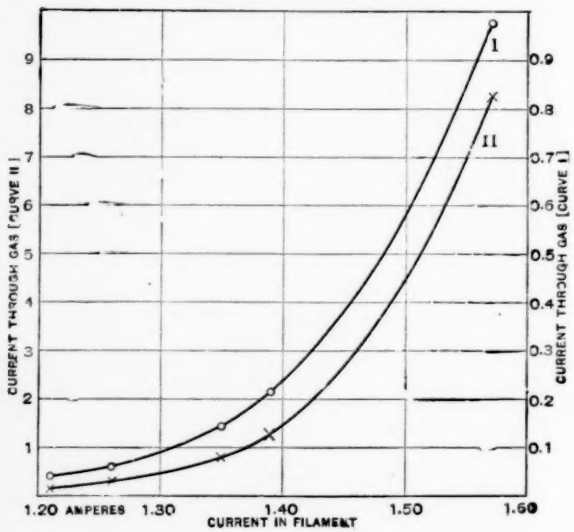

Fig. 2.

owing to the small bore of its graduated tube. The gauge seemed to be especially sensitive to the sources of error to which all gauges of this type are susceptible. But the readings give an indication of the relative values of the pressure in different experiments. The gauge was also useful in indicating the degree of constancy of the pressure during the progress of a series of measurements.

Experiments at High Vacua.

The character of the results obtained at high vacua is shown by the curves given in Fig. 3. In these curves the abscissas show the p. d. between filament and cylinder, while the ordinates give the cor-

responding values of the current. Each curve corresponds to a certain constant value of the current in the lamp, *i. e.*, to a constant temperature of the filament. During the determination of these curves the pressure as given by the gauge varied between .003 mm.

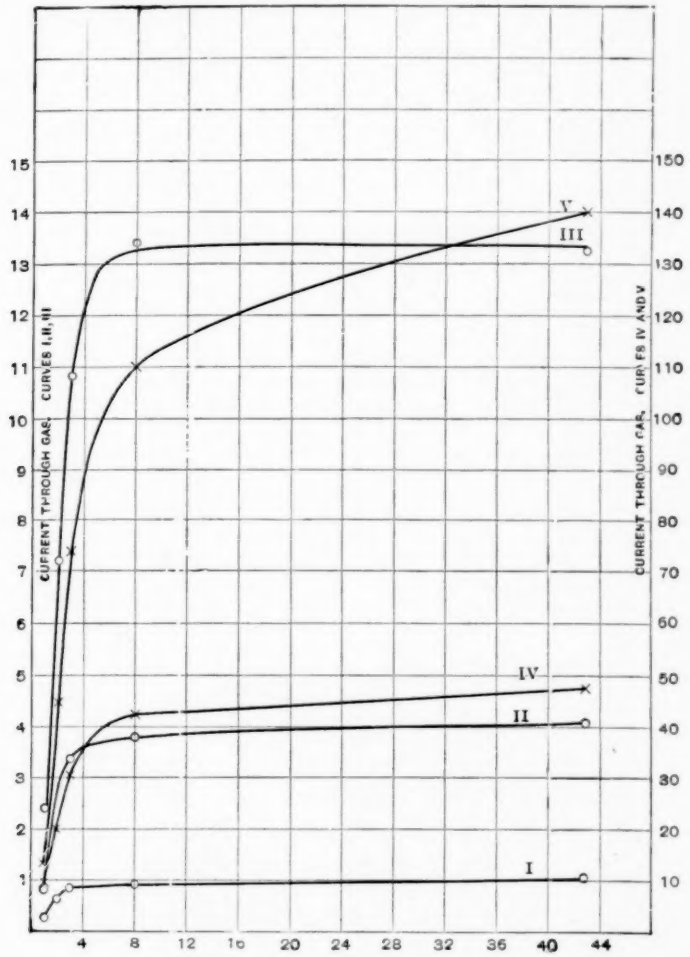

Fig. 3.

The unit in the case of the abscissas of these curves is the e. m. f. of one Leclanché cell — approximately 1.4 volts.

The curves were taken with the following values of the current in the filament.

Curve I. 1.2 amp.
Curve II. 1.3 amp.
Curve III. 1.4 amp.
Curve IV. 1.52 amp.
Curve V. 1.6 amp.

Pressure, 0.003–0.007 mm.

and .007 mm. This variation was without appreciable influence on the results.

It will be noticed that the curves plotted in Fig. 3 show the characteristics that we should naturally expect in the case of gaseous conduction. The current at first increases approximately in proportion to the potential difference. A limit is soon reached, however, beyond which a further increase in the p. d. causes only a slight increase in the current; "saturation" has been reached. Saturation is more complete in the curves corresponding to low lamp currents, *i. e.*, to a relatively low temperature of the filament.

To some extent these curves are misleading. Under the conditions of the experiment very similar curves would be expected even if the galvanometer current did not depend at all upon the magnitude of the p. d. between filament and cylinder, but only upon its sign.

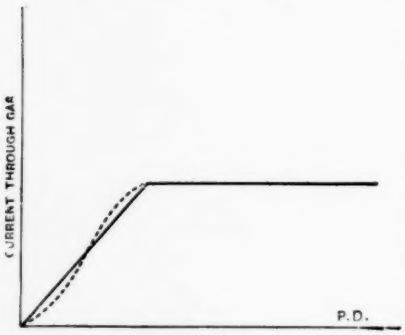

Fig. 4.

It will be remembered that the galvanometer circuit was connected to the negative terminal of the filament, and that the current in the filament was sufficient to maintain a p. d. of from 5 to 8 volts between its terminals. At the beginning of each experiment, therefore, with no e. m. f. in the galvanometer circuit, all portions of the filament are positive with reference to the cylinder. Since it is only when the incandescent filament is a kathode that the Edison effect is observed, no current will flow. If a small e.m.f. is placed in the galvanometer circuit so as to make the cylinder positive, a portion of the filament near its negative end will now be at lower potential than the surrounding cylinder, and current can flow. As the potential of the cylinder is raised, a greater proportion of the filament becomes negative with reference to it, and can therefore take part in the transmission of current. When the potential of the cylinder is equal to, or greater than, the potential of the positive end of the filament, the whole incandescent surface has become kathode. If we assume that the current is in proportion to the kathode surface,

and that the fall of potential through the filament is uniform, we should expect the form of curve shown by the full line in Fig. 4, regardless of any direct dependence of current on p. d. Since the ends of the filament are cooled a little by conduction, the curve to be expected would be rounded somewhat as shown in the dotted line. This is exactly the form of curve obtained experimentally.

The important influence on the results of the difference of potential between different parts of the filament is well brought out in Fig. 5. The curves were taken under identical circumstances

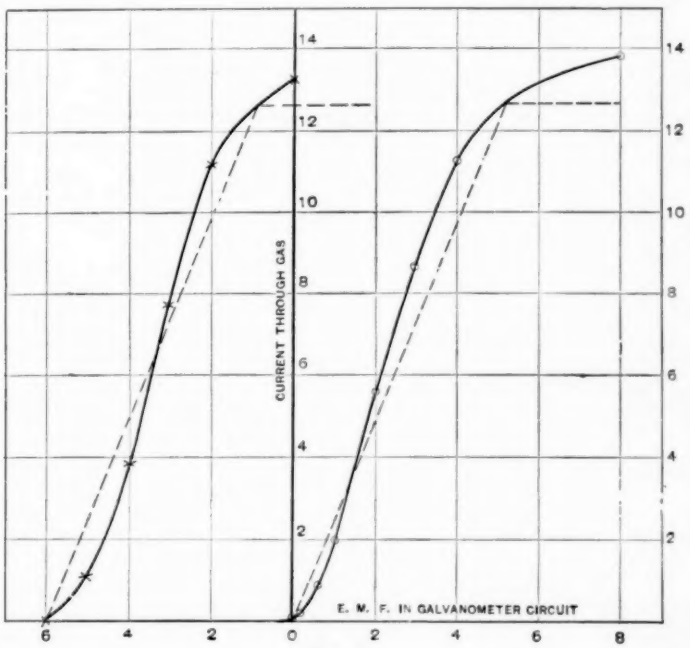

Fig. 5.

except that in one case the galvanometer circuit was connected to the negative terminal of the lamp filament, as usual, while in the other case it was connected to the positive terminal and the e.m.f. reversed. The character of both curves is readily explained then by the variation in the amount of the filament that was the kathode with respect to the cylinder.

In Fig. 5 the curves to be expected from the known p.d. between the lamp terminals upon the assumption of a uniform drop through the filament are shown as dotted lines.

These curves, and others like them, show that the increase in current is a little more rapid, and continues longer, than could be accounted for by increase in kathode surface only. The current does depend somewhat upon the p.d. between filament and cylinder. But this dependence is slight. The number of negative ions developed at the incandescent surface is therefore increased very little by increasing the potential gradient. Lenard found that the behavior in the case of the photo-electric discharge at low pressures is similar to this, for the discharge current for a given illumination was changed very little by changes in e.m.f. It appears that the development of these negative ions, either in the case of an illuminated surface or an incandescent one, is more largely dependent upon some phenomena occurring at the surface itself than upon any electrical influences from outside.

It will be observed that there is nothing in the results obtained at high vacua to suggest that the phenomena are influenced by the small trace of gas left in the tube.

EXPERIMENTS AT LOW VACUA.

A series of curves taken under the same conditions as before except that the pressure in the tube was much higher had the same general form at low voltages as the curves shown in Fig. 3. In each case practically complete saturation was apparently reached at 10 or 11 volts. But when a larger potential was employed the current was found to be much larger than the saturation value. In Fig. 6 are shown three such curves. The vertical scale is different for each curve. The pressure in these cases was about 0.5 mm. Similar curves were obtained at a pressure of 0.16 mm., except that for a very low temperature of the filament (current in filament = 1.19 amperes) no abnormal increase in current was observed, even with an e.m.f. of 60 volts.

It was in these observations that what we have called the "creep" was first noticed. For low e.m.f.s the galvanometer promptly took a steady deflection; but when an e.m.f. greater than about 10 volts was employed the galvanometer current did not reach a steady value at once, but slowly increased for some minutes.

Curve I. in Fig. 7 shows the manner in which this change took place. The ordinates are proportional to the galvanometer deflec-

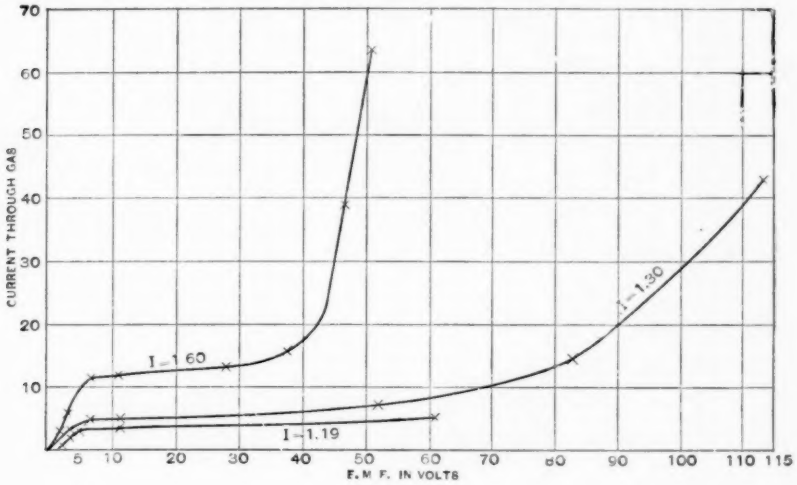

Fig. 6.

tions and the abscissas to time. In this case the current through the lamp filament was 1.50 amperes and the e.m.f. in the galva-

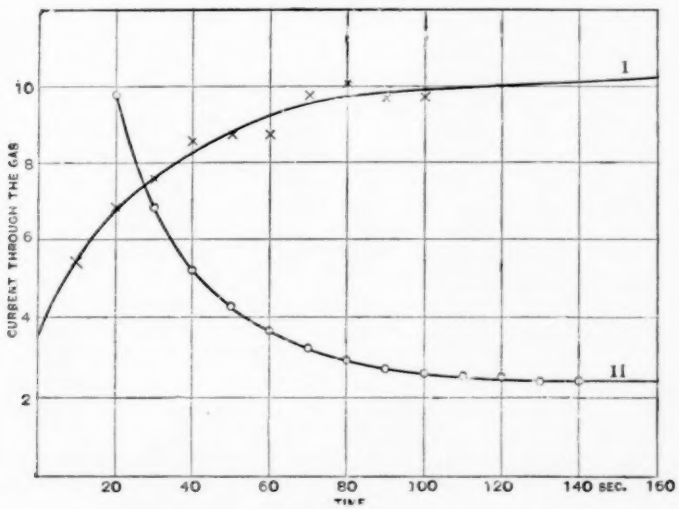

Fig. 7.

nometer circuit was 60 volts. As shown in the curve, readings were taken every ten seconds. Even at the end of 100 seconds the cur-

rent had not attained a steady value. Cases were noted where it required a much longer time than this for the current to reach its maximum value; in one instance an increase was observed for fifteen minutes after closing the galvanometer circuit. But for lamp currents greater than 1.74 amperes the galvanometer current, for all values of the difference of potential, reached its maximum apparently at once; certainly in a time too short to be observed with the galvanometer.

The data used in the curves of Fig. 6 were the values obtained after the deflections had become steady. The first deflection for a p. d. greater than 10 volts was in general approximately the same as that given by 10 volts; but the current increased slowly up to the value shown in the curves of Fig. 6.

If a rather high difference of potential between the filament and the cylinder was maintained for some time and then suddenly decreased, the reverse effect was obtained. For example, if the galvanometer circuit had been closed for some time with 60 volts, and if the e.m.f. was then suddenly decreased to 10 volts, the galvanometer current would at first be much larger than that corresponding to an e.m.f. of 10 volts as shown on the curve. It would, however, slowly decrease until it reached the value that would have been obtained had the higher e.m.f. not been used. Curve II. in Fig. 7 is a curve plotted from the observed values of the current at different times after the e.m.f. had been suddenly decreased from 60 volts to 11 volts. In this case the lamp current was 1.75 amp.—a value so great that the time *increase* could not be detected on applying the higher e.m.f.s. This slow decrease in the current was always noticed on suddenly changing to any lower p. d. from a p. d. greater than the one corresponding to the point of inflection of a curve like one of those in Fig. 6. Although the "creep down" was observed for the higher lamp currents, the duration of it was less than in the cases where the values of the lamp current were those corresponding to cases that gave the "creep up."

The rate of decrease of the galvanometer current on changing from a higher to a lower e.m.f. was to a large extent independent of other changes in the connections. For example the decrease for

a given time after the higher e.m.f. was cut out was practically the same whether (1) the lower e.m.f. had been connected all the intervening time or (2) the circuit had remained open, up to this time, or (3) the cylinder had been connected directly to the filament.

Discussion.

It is clear from the experiments here described that the phenomena are much simpler at high vacua. The incandescent filament still discharged negative electricity, even in the highest vacuum that our pump could give. It seems probable, therefore, that the electrons by which the charge was carried come from the incandescent body, rather than from gases in contact with it.

The resemblance between the behavior of an incandescent kathode and an illuminated kathode, already referred to at the beginning of this article, is too striking to be overlooked. In each case the phenomena are primarily due to the emission of negative electrons by the surface in question. In one case it is light — more especially ultra-violet light — that acts as the agent to bringing about the emission of such electrons. In the other case the high temperature of the surface produces the same result. The number of negative particles sent off in unit time depends in one case upon the intensity of the illumination; in the other case upon the temperature of the surface. In neither case is it largely influenced by the electric field, provided only that the direction of the field is such as to make the electrode a kathode.

At high vacua the kathode ray particles are the sole carriers of the current, and since their number depends chiefly on the degree of incandescence the curves show saturation at low values of the e.m.f. As the pressure is increased the air surrounding the kathode becomes an important factor. As the p. d. is increased the kathode rays will move at an increasing speed, and will finally move fast enough so that upon colliding with the molecules of gas these will be ionized. The ions thus formed will participate in the conduction between the filament and the cylinder, so that the current will again increase with the e.m.f. An explanation is thus offered of the form of the curves shown in Fig. 6. Similar curves have been observed for the photo-electric discharge by several observers.

[1] Kreusler, Beibl., 22, p. 698; v. Schweidler, Wien. Sitzungsber., 107, p. 881 and 108, p. 273, 1898; also Beibl., 23, pp. 513 and 585.

Townsend has also observed a somewhat analogous phenomenon in the case of the conduction through gases that have been ionized by the action of Röntgen rays.[1] The effects observed by him were satisfactorily explained upon the assumption that the negative ions produced were able to ionize other portions of the gas, just as kathode rays do, provided that the conditions were such as to enable them to acquire a certain minimum speed between collisions. Curves similar to those shown in Fig. 6 were also obtained by McClelland[2] in the case of the current passing between a hot platinum kathode and a cold anode.

Although the hypothesis of ionization by collision at first appears to afford an entirely satisfactory explanation of the results obtained at low vacua, yet the various peculiarities of the "creep," and especially its long duration, offer serious objections to this explanation. Many attempts were made to throw further light on this phenomenon. If it were merely a case of a slow change in the ionization in the gas, the conductivity might be expected to last a short time after the circuit through the filament was broken. But the galvanometer current dropped at once to zero when the lamp circuit was broken. Yet if the lamp circuit was at once closed again it was found that the increased conductivity was still present. Suddenly reversing the direction of the e.m.f. in the galvanometer circuit also brought the needle promptly to a vibration about its zero.

An e.m.f. of 60 or 80 volts was applied long enough to obtain a steady current, and the galvanometer circuit was then broken for a definite time, from 10 to 30 seconds. On closing the circuit the galvanometer current was found to be somewhat less than its steady or maximum value, but still much greater than if the circuit had not previously been closed. The current would creep up to the value it had before the circuit was broken; but the fact that it was initially larger indicated that the increased conductivity of the tube had in part remained during the time the circuit was broken.

Reversing the e.m.f. in the galvanometer circuit for 20 seconds had no appreciably different effect from that of merely breaking the

[1] Phil. Mag., Series 6, Vol. I, p. 198. 1901.

[2] Cambridge Philosophical Proceedings, Vol. 11, p. 296, 1902.

galvanometer circuit for the same time. The conductivity decreased apparently less if the lamp circuit alone was broken than if the galvanometer circuit alone was broken, *i. e.*, after keeping the lamp circuit open for 20 seconds the galvanometer current was larger than if only the galvanometer circuit had been broken for the same length of time. Breaking both circuits simultaneously for 30 seconds apparently produced even less diminution of the increased conductivity than breaking the lamp circuit alone.

These observations show that the gas in the tube does not remain a conductor for any considerable length of time. Even if a current too small to deflect the galvanometer should flow, the gas would rapidly lose its ions and its conductivity. Yet, as shown, the increased conductivity is affected little, if any, by breaking the lamp circuit.

It therefore appears that the creep cannot be explained simply by the ionization of the residual gas. We are led to suggest that the effect is due rather to some gradual change in the incandescent surface, this change being of such a character as to increase the rate at which kathode rays are produced; when the filament is no longer negatively charged, or when it is allowed to become cold, the surface gradually returns to its original condition. In forming any opinion regarding the nature of this assumed change in the incandescent surface we must bear in mind the fact that it occurs only at low vacua; at high vacua nothing abnormal was observed. It is therefore clear that the presence of an appreciable amount of residual gas is essential. The temporary nature of the change in question shows that it cannot be due to oxidation of the carbon surface, or to any similar chemical action. But the results may be satisfactorily explained if we assume that something is deposited on the surface without uniting with it chemically. The material deposited must have a positive charge, for it is only when the filament is negative that the effect is observed.

When considered from this point of view the phenomena of the creep effect have many points of similarity with cases of excited radio-activity. One is tempted to believe that an active gas is liberated by the hot carbon and that the negatively charged filament acquires temporary radio-activity in the same way that it would do if surrounded

by the emanations of radium or thorium. There is, however, one peculiarity in the effect which shows this explanation to be untenable. The change in the filament which caused the creep persisted for several minutes, even if the current in the filament was broken; but no current could be detected in the galvanometer while the filament was cold. It seems, therefore, that the material deposited on the filament does not possess in itself any marked power of ionizing the gas; it merely intensifies the effect of incandescence. The phenomena are more satisfactorily explained if we assume that the deposit on the filament produces a condition analogous to polarization in a voltaic cell. If the positive ions attracted to the surface experience a certain difficulty in giving up their charges, a double layer will be formed at the surface, positive just outside and negative within. This will greatly strengthen the electric field at the surface, so that the tendency for negative electrons to escape from the carbon will be increased. It seems surprising that such a double layer is not at once destroyed when the e.m.f. is reversed. But the porous character of the carbon probably permits the positive ions to penetrate to a considerable depth, so that they cannot be quickly removed even by a reversed field. For the same reason the polarized condition requires an appreciable time for its complete development.

It is hardly to be expected that the negatively charged filament itself can be the source of the positive ions which cause this polarization; these come rather from the residual gas. It is evident therefore that the effect can only occur at relatively high pressures. The formation of the assumed layer, and the consequent "creep" effect, can only take place when positive ions are present in the gas. But there is nothing to produce ionization except the kathode rays from the hot carbon; hence we cannot expect the polarized layer to form until the velocity of the kathode rays is sufficient to produce ionization in the surrounding gas. In other words the creep effect will only be observed when the e.m.f. in the galvanometer circuit exceeds a certain critical value. For e.m.f.'s less than this we have the ordinary phenomena of conduction in a gas with ionization at the negative terminal only; the gas itself has no influence upon the conduction, except so far as it alters the speed of

the negative ions. No positive ions are present. For higher electromotive forces the phenomena are modified in two ways: (1) The kathode rays produce ionization in the air, which now participates in the conduction; (2) positive ions now being present, some of these collect on the carbon filament and form there a positively charged layer. This increases the ease with which negative electrons escape from the hot carbon, so that kathode rays are produced in greater intensity. When the critical value of the electromotive force is exceeded there are therefore two reasons for an increase in current, and the sudden bend in the curves of Fig. 6 is a natural result.

Since the MS. of the preceding article was sent to the printer, a preliminary paper has been published by J. Stark, which deals in part with experiments upon the same subject.[1] The arrangement of apparatus in these experiments was somewhat different from that adopted by us; and to this fact is doubtless to be ascribed a part of the difference in the results obtained. But we are unable to account for all of the differences in this way.

The filament employed in Dr. Stark's experiments required a potential difference of 6 volts at its terminals to bring it to the proper incandescence. This is approximately the same p. d. that was used in our own work. Instead of a cylindrical anode, however, a wire was used, this being placed in a side tube at a distance of 6 mm. from the filament. At pressures ranging from 0.029 mm. to 4.2 mm. the curves connecting the e.m.f. and the current through the gas show a general resemblance to those plotted in Fig. 6 on p. 247, but differ from them in three ways, viz.: (1) the rapid increase in conductivity, which indicates the beginning of ionization by collision, begins at about 11 volts, instead of at about 30 volts; (2) a second more rapid rise in the conductivity of the gas occurs at about 27 volts; (3) at about 40 volts the current reaches a new saturation value, far greater however, than the value corresponding to e.m.f.s below 10 volts.

The sudden increase in conductivity observed by us seems to correspond to the *second* break in the curve obtained by Stark. That this point corresponds to the beginning of ionization by collision in

[1] Physikalische Zeitschrift, Vol. 5, p. 51, Jan. 15, 1904.

the nitrogen remaining in the tube, as suggested by Stark, seems to us quite probable. In fact this is practically the explanation reached by us. The third stage mentioned by Dr. Stark, corresponding to potentials higher than 40 volts, was not observed in our experiments; there was nothing to suggest renewed saturation, even at potentials as high as 115 volts. This may perhaps be due to differences in the form and dimensions of the apparatus. We are at a loss, however, to account for the absence in our curves of the first stage mentioned by Dr. Stark. No sudden increase in conducting power in the neighborhood of 11 volts was observed in our experiments. Dr. Stark explains the break in his curves at about 11 volts as due to the ionization of mercury vapor, and therefore sets the ionizing potential for mercury vapor at 11 volts. Since the tube used in our experiments remained in connection with the pump for days at a time there is every reason to believe that mercury vapor was present. Yet there is no indication that it participated in the conduction in any way. This is true not only in the case of the observations at the relatively high pressure of 0.5 mm. (Fig. 6) but also in the case of the observations at low pressures (.003 mm.) shown in Fig. 3. The latter observations seem especially well adapted to show the influence of the mercury vapor if such influence exists, for the disturbing effect of other gases is reduced to a minimum. Unfortunately the observations of Dr. Stark do not extend to pressures below 0.029 so that no direct comparison of our results is possible.

In this connection the great difficulty that is always experienced in completely freeing a carbon filament from occluded gases should not be forgotten. In order to accomplish this so far as possible we were in the habit of running the filament during the process of exhaustion at a higher temperature than that used when making measurements. Such a procedure was indeed absolutely necessary in the work at low pressures, since the liberation of gas by the filament made it otherwise impossible to maintain a high vacuum. When it is remembered that some liberation of gas will still occur even when this precaution is taken, and that it will occur with greater rapidity when the temperature of the filament is high, certain peculiarities in the curves shown in Fig. 3 become significant. Curves I., II. and

III. in this figure show practically complete saturation, with no indication of ionization by collision. There are the curves corresponding to small lamp currents and therefore low temperatures. In Curves IV. and V., for which the temperature of the filament was higher, the saturation is not so complete. May it not be that we have to deal here with ionization of the gas liberated from the filament? And may it not be that the effects ascribed by Dr. Stark to mercury vapor were in reality due to some readily ionized gas given off by the filament?

Dr. Stark does not appear to have observed the phenomenon referred to by us as the "creep." It seems probable that the filament was kept at so high a temperature in his experiments that steady deflections were reached almost instantly.

FIBERS RESEMBLING FUSED QUARTZ IN THEIR ELASTIC PROPERTIES.

BY K. E. GUTHE.

THE advantages of quartz fibers as suspensions, principally their small elastic fatigue, great strength and the possibility of drawing very fine threads, have led to their use in a great many important investigations where fine suspensions and a steady zero-point are required.

In my work suspensions were needed able to carry a load of 500 g. and more, and at the same time having as small an elastic fatigue as possible. Naturally fused quartz was selected as the substance promising the best results. Such fibers must be rather thick, from 0.1 to 0.2 mm. in diameter. Since it was desired to draw them at least one meter in length, it required the melting of a rather large bead free of air bubbles. The process of building up quartz rods is known to be rather tedious. Shenstone's method [1] is simpler and consists of heating the crystals to about 1000° C. and suddenly quenching them in cold water. The crystals after such treatment are not shattered and do not break when brought into the oxy-hydrogen flame. A stick of the proper dimensions is easily formed, containing however a large number of air bubbles. To remove these is a very tedious task and particularly exasperating, because quartz at the high temperature of the blowpipe flame is quite volatile, about one half of the mass evaporating during the process.

Whichever method is employed it means a considerable loss of time, if a great many fibers of the dimensions necessary for my work have to be drawn.

Boys [2] in his first paper on this subject mentions his experiments on a great many minerals, of which only a few, however, could be

[1] Shenstone, Nature, Vol. 64, p. 65, 1901.

[2] Boys, Phil. Mag., Vol. 23, p. 489, 1887.

drawn into fibers, and these, he says, were far inferior to those made from fused quartz. I have also tried a great number of substances, all silicates of magnesium : Enstatite, olivine, serpentine and meerschaum, but without success. Only the amphibol-asbestos, $Mg_3Ca Si_4O_{12}$, in the ordinary mineral form as well as when specially prepared for chemical purposes, and the steatite or soapstone, $Mg_3H_2 Si_4O_{12}$, give clear beads before the oxy-hydrogen blowpipe. These under proper precautions can be drawn easily to fibers of the desired dimensions. The soapstone is especially easily worked and gives fibers of practically the same elastic properties as those of quartz. Since the method of making them is so simple and requires little time, a short description may be of general interest.

The soapstone, when heated to a high temperature, becomes exceedingly hard, and is used commercially under the name of "lava" for making apparatus intended to be able to stand high temperatures, for example, tips of gas burners. While it cannot be melted before an ordinary blowpipe, it does so before an illuminating gas-oxygen jet and forms a clear bead, usually of a greenish tint due to the presence of a trace of iron. The original soapstone or the "lava" will do equally well, the most convenient form being small cylindrical sticks.[1] If the flame of the blowpipe is too long or the mixture of the gases not well adjusted, the substance will boil violently, while with a small, quiet flame it melts without boiling, a slight development of gases being noticeable only at the upper, cooler part of the bead. After taking the pearl out of the flame, the thread is drawn, its thickness depending upon the temperature and rapidity with which it is drawn. Very fine fibers can thus be secured. The fibers should not be heated after being drawn. In a Bunsen burner, for instance, they will immediately become white and break to pieces.

Elastic Fatigue. — It is very well known that fine quartz fibers show hardly any elastic fatigue,[2] but with thicker fibers of 0.1 to 0.2 mm. diameter, the effect of elastic fatigue became quite apparent. Careful experiments showed that small twists, or twists of small

[1] Such cylinders 3 mm. in diameter and 7 cm. long, giving colorless transparent beads about 5 mm. in diameter, were obtained from the Chattanooga Sunlight Lava Manufacturing Company.

[2] Boys, l. c., p. 496 ; Threlfall, Phil. Mag., Vol. 30, p. 113, 1890.

amplitude do not affect the zero-point, while larger twists being continued for several minutes would displace the zero-point. The apparatus used allowed a turning of the torsion head while the lower end of the fiber was held in its original position. After release the zero could be redetermined in a few seconds and the slow disappearance of the effect of elastic fatigue observed. With fibers 70 cm. long and 0.1 mm. diameter, the displacement after release from a torsion of 360° lasting 5 minutes, amounted to as much as 2 in 3,000. Fibers made of steatite showed about the same effect, which is however very much smaller than any other substance experimented with, and about one half to one third of that shown by steel or phosphor bronze.

For these experiments the quartz and steatite fibers had been silvered at their ends, then coppered and finally soldered into large brass pins clamped securely in the supports. Thus the elastic fatigue observed cannot be attributed to an effect of the material with which fibers were fastened to the other parts of the instruments.

Tensile Strength. — The tensile strength of very thin quartz fibers is quite large: 10×10^9 dynes per cm.^{2}, but it decreases appreciably with increasing thickness.[1]

The following experiments show the same result. From Table I. it is seen that the tensile strength of the steatite fibers is at least as large as that of quartz. For comparison the values given by Boys, (B), are added. The decrease of tensile strength for the

TABLE I.

Quartz.		Lava.	
Diameter.	Tensile Strength.	Diameter.	Tensile Strength.
(B) 0.00048 cm.	11.5×10^9 dynes / cm.^{2}		
(B) 0.00175	8.0		
0.0075	6.21	0.006 cm.	13×10^9 dynes / cm.^{2}
0.0095	6.91	0.0095	9.96
0.010	6.16	0.010	9.11
0.015	4.5	0.011	7.19
0.0175	3.91	0.0135	6.92
0.018	4.08		

[1] Boys, Phil. Mag., Vol. 30, p. 116, 1890.

thicker fibers is very striking. For the thickest fibers it is, however, still as large as that of brass.

Coefficient of Simple Rigidity. — The torsional coefficient was determined by the well-known method of vibration. The diameter and length of the fibers were carefully measured, a cylinder made of Tobin bronze was suspended by the fibers and the period of the torsional vibrations determined by means of a chronometer. Then

$$n = \frac{8\pi IL}{r^4 T^2}.$$

Since it became apparent that different fibers of the same substance gave slightly different values, and since the diameters of the fibers were measured only with an accurate micrometer, no correction for the coefficient of expansion was made.[1] The mean of a number of experiments gave for quartz $n = 3.4 \times 10^{11}$; for the steatite fiber a value from 2.6 to 4.2×10^{11}. Also in this respect the new fiber almost exactly equals fused quartz. The temperature coefficient of the torsional constant of quartz is given by Threlfall as + 0.000133; by Barnett[2] as + 0.000115; I obtained + 0.000149. Steatite does not show the remarkable property of becoming more rigid at higher temperatures. Its temperature coefficient is negative, $\alpha = -0.000193$. The results obtained are given in Table II. and plotted in Fig. 1.

TABLE II.

Quartz.			Lava.		
Temp.	Period.	n	Temp.	Period.	n
8°.5	8.3083	3.4044×10^{11}	11°.45	8.7507	3.9830×10^{11}
19°.0	8.3020	3.4096	17°.10	8.7556	3.9777
22°.45	8.2997	3.4115	22°.05	8.7590	3.9755
26°.2	8.2973	3.4134	26°.75	8.7625	3.9723

Coefficient of Expansion of Fused Steatite. — The following experiments were made by Mr. L. G. Hoxton, of the Bureau of Standards, using a rather thick fiber of fused steatite.

The length between two fairly well defined marks 110.4 mm.

[1] See also Threlfall, l. c., p. 108.

[2] Barnett, PHYS. REV., Vol. 6, p. 119, 1898.

apart was compared at different temperatures with that of the standard nickel steel decimeter No. 43, belonging to the Bureau and furnished with a certificate from the Bureau International des Poids et Mesures, which gives its linear coefficient as $\alpha =$

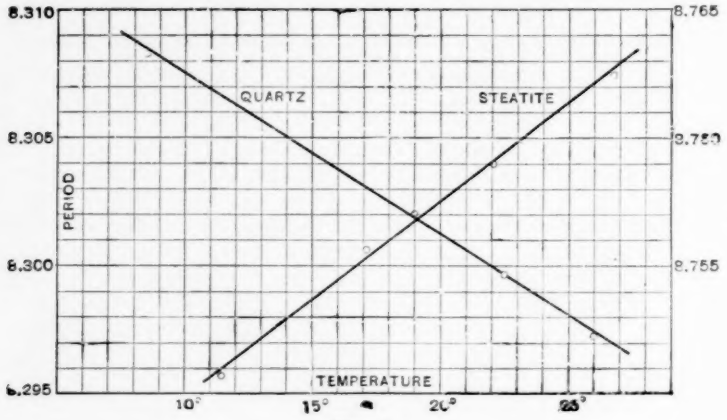

Fig. 1.

0.96×10^{-6}. In the following table the differences between the length of the fiber and the standard are marked F—NS. In the third column is given the linear coefficient of expansion counting from the lowest temperature and in the last the number of observations made for each temperature.

TABLE III.

Temp.	F—NS.	α	No.
+ 19°.31	+ 10.3 μ	0.0_5317	5
+ 3°.56	6.4	0.0_558	7
− 2°.08	2.8		5
+ 21°.8	12.4	0.0_5365	4

The average relative coefficient is found from the normal equation

$$1059.4\, x = 410.1\, \tfrac{1}{110400}$$

$$\therefore \quad x = 0.00000350.$$

The absolute linear coefficient of expansion is therefore

$$\alpha = 0.0000045.$$

The comparisons were made on a Zeiss horizontal comparator, which allows a direct reading of 1.0 μ.

An attempt to obtain a fiber with a zero temperature coefficient by melting quartz and steatite together was unsuccessful. As soon as too much quartz is present, the bead loses its transparency. If beads made of soapstone should not be clear, it is generally due to an excess of quartz in the substance. In such a case the addition of a small amount of magnesia or magnesium carbonate will clear up the bead.

Index of Refraction of Fused Steatite. — Two small prisms were ground from steatite beads and their index of refraction for sodium light determined by the method of minimum deviation. The instrument used was a spectrometer reading to half minutes. Table IV. contains the results.

TABLE IV.

	Angle of Prisms.	Min. Deviation.	n
Prism 1	60°3′	44°1′	1.576
Prism 2	60°4′	44°17′	1.578

Also for these determinations I am indebted to Mr. L. G. Hoxton.

From the above comparison we conclude that we have found in fused steatite a substance which shows all the characteristic properties of fused quartz. The method of drawing fibers from it is very simple, and since the substance is easily secured (an old jet of a gas burner will furnish a large number of fibers) these fibers may with advantage be used, where economy of time is an important question. One more advantage lies in the easy handling of the fiber. While thick quartz threads break easily when bent, those of steatite may be bent considerably more without breaking.

Barnett[1] working with fibers from 0.005 cm. to 0.007 cm. in diameter did not detect any time effect upon the period. But a distinct effect was found by me with either quartz or steatite fiber. The quartz fibers became more rigid in course of time, a behavior resembling that of metals.[2] Professor Carhart and the writer have found, however, a decrease of rigidity in the case of phosphor bronze.[3] Con-

[1] Barnett, l. c., p. 116.

[2] Nichols and Franklin, El. of Phys., I., p. 99.

[3] Carhart and Guthe, PHYS. REV., Vol. 9, p. 292, 1899.

tinued vibrations, especially of very large amplitude, seem to have a distinct influence upon this "settling down" to a steady state, if such can be obtained. Investigations are carried on concerning this time change, in order to determine whether different specimens of quartz will differ in their behavior — ours contained quite a large amount of strontium — or whether the difficulties are to be attributed simply to the size of the fibers.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE CONTINUOUS METHOD OF STEAM CALORIMETRY.

By Joseph H. Hart.

THE continuous method of steam calorimetry was devised as the result of an attempt to measure the variation of the specific heat of a body in a magnetic field. Nearly all the known calorimetric methods were used and the difficulty of getting concordant results in consecutive experiments beyond the third significant figure without special refinement in the method and in the observations was early recognized. The present method while inapplicable to that problem was thereupon developed as capable of a much greater accuracy with less refinement in the method than any method hitherto employed.

If a stream of water at a temperature θ_1 is passed through a worm which is immersed in a steam bath and emerges at a temperature θ_2 the heat absorbed by the water is $mS(\theta_2 - \theta_1)$ where m is the mass of water passed through and S the mean specific heat between θ_2 and θ_1. If all the heat absorbed is obtained by direct condensation of steam alone, we have the equation,

$$ML = mS(\theta_2 - \theta_1),$$

where M is the mass of the condensed water and L the latent heat of steam.

In the practical development of the method the process was made continuous. The condensed water was collected and drawn off continuously. This mass and the mass of the water which had passed through the worm were measured some time after an experiment so that complete attention could not be given to the determination of the temperature differences. Thus L or S could be readily determined, if the other was taken as known. Barnes' values of the specific heat of water were taken and a large number of experiments were made to determine L in order to test the efficiency of the method.

Many difficulties were encountered in the preliminary tests on account of the radiation and conduction from the walls of the calorimeter into the worm and many of the small details of construction in the worm and calorimeter are the result of much study. The measurement of the condensed water also presented difficulties which, it is believed, have been overcome. The construction given below eliminates the radiation and conduction of heat into the worm from the calorimeter, or at least satisfactorily accounts for it in the quantity of condensed water collected, both by theory and practice.

The prime requirement in the development of this method was the uniform production of a large amount of dry steam at constant pressure. The only way by which the temperature θ_2 could be kept constant, when the steam supply varied, was by regulating the velocity of the flow of the water (m) through the worm. This was found undesirable and a number of experiments were made to construct a satisfactory boiler. A slight change in the water level in the boiler was found to produce considerable change in the steam supply. This became at once evident by a change in the temperature θ_2, if all the steam supply was being condensed. Boiling by bumping also produced changes and external drafts on the boiler had an appreciable effect. The slightest complication in the construction of the boiler also produced considerable variations in the pressure. The boiler used in the final experiments gave very satisfactory results and is shown in Fig. 1. It is completely covered with an asbestos jacket, and is connected through a small intermediate compartment with an adjustable constant level apparatus. In the top is a water manometer with constant zero resulting from condensation of steam. Its dimensions are 40 cm. by 15 cm. by 25 cm. high. It was connected to the calorimeter almost immediately through a tube 10 cm. in diameter. This boiler was used for stretches of over fourteen hours without any attention whatever and with no appreciable change in the steam supply or in the pressure. The water manometer was very sensitive and showed small variations in the pressure when the calorimeter was connected to the boiler. When the condensation commenced, however, this variation diminished and the pressure could be made approximately constant throughout an experiment, after the rate of condensation

had become constant, by varying either the heat supply or the size of the outflow tube from the calorimeter to the condenser. This steam pressure, constant throughout an experiment, was less than 0.5 mm. water pressure and was sometimes added to the barometric reading, but was more often totally neglected.

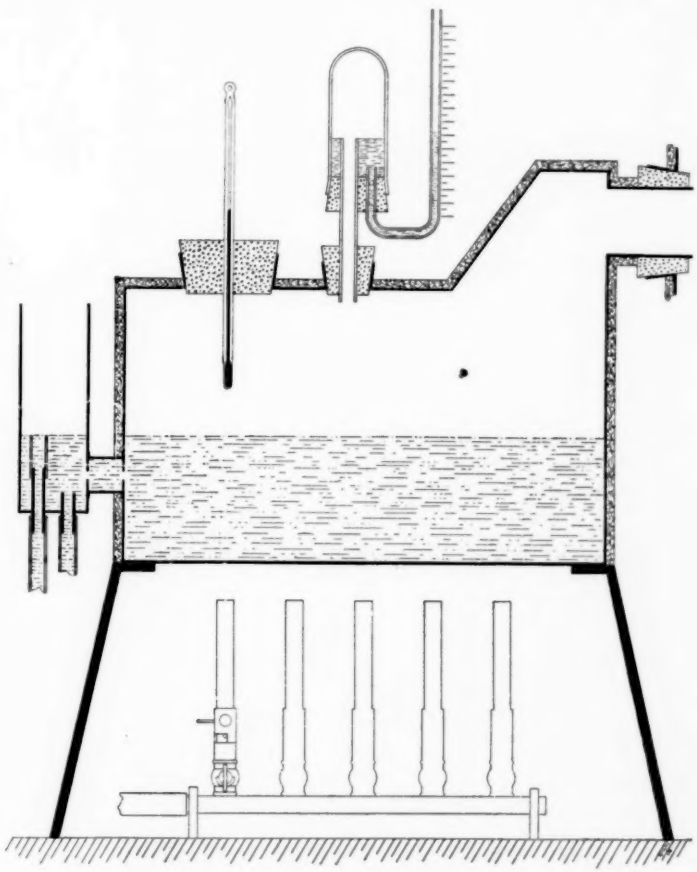

Fig. 1.

The calorimeter was a metal box 20 cm. by 20 cm. by 75 cm. high with a conical top and had an inlet and an outlet through large corks for the water (*m*), an inlet and an outlet for the steam and an outlet for the water condensed on the worm. The calorimeter was covered with a heavy asbestos jacket which prevented much con-

densation on the walls of the calorimeter. A cross section of the calorimeter and worm is given in Fig. 2. The worm was made of the thinnest sheet copper compatible with the necessary strength and consisted of a cylinder 3 cm. in diameter and 60 cm. long with

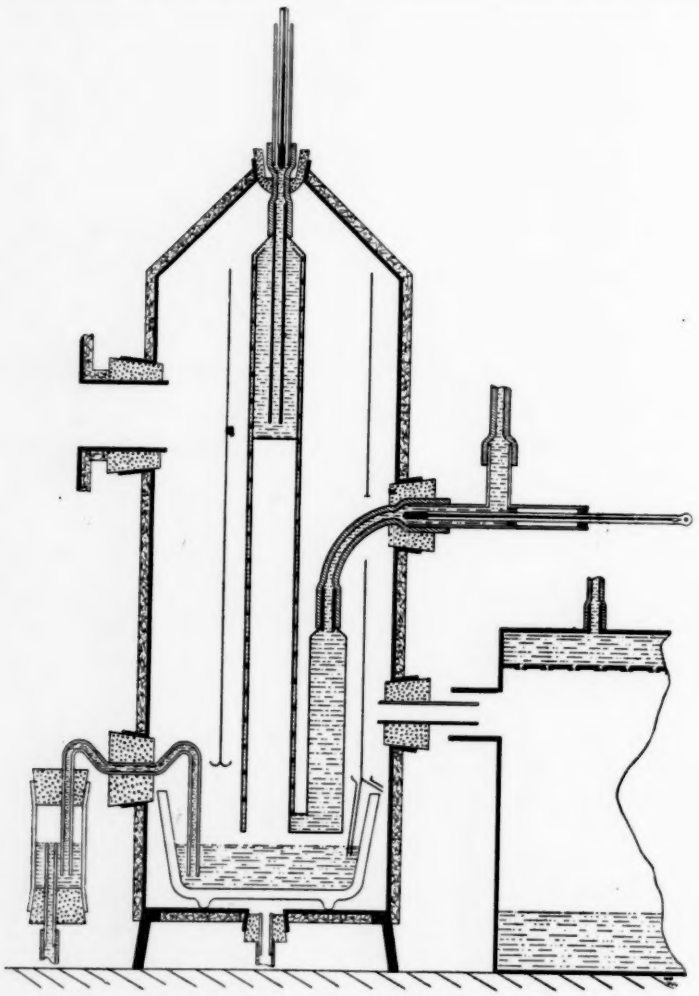

Fig. 2.

a partition across it one quarter of the way from the upper end. This cylinder was then wrapped spirally with a thin strip of copper and another tight-fitting cylinder of very thin copper sheet was screwed on this, thus forming a worm, like a very long thin ribbon.

This worm was connected at the lower end with a reservoir of thin sheet copper 3 cm. in diameter and 15 cm. in height. The partition across the inner cylinder formed the base of the incoming cold water reservoir, which had for its sides and top the ribbon-like worm filled with warm water. The incoming water was carried by a tube to the bottom of this reservoir.

The object of this construction of the worm was to present a large heating surface for the steam, to have a large quantity of water heating constantly, and to keep the incoming cold water from close thermal contact with the steam, thus preventing a rapid local condensation of the steam. While a rapid local condensation of steam is undesirable, since the rapid cooling would tend to lower the temperature of the condensed water below the temperature of condensation, it is not necessarily detrimental to the method. For the condensed water, whatever its temperature may be soon after condensation, if it remains a sufficient time in the calorimeter, will in turn condense sufficient steam to raise it to the temperature of condensation.

The cold water was sent into the worm through a heavy rubber tube which was sealed, steam-tight, into a large cork at the apex of the conical top of the calorimeter. An air chamber surrounded a portion of the tube in the interior of the cork. The walls of the cork were cut at an angle so that the water which was condensed from the steam as a result of conduction through the cork to the cold water was collected with the water condensed on the worm. A cross section of this entrance together with the position of the thermometer is given in Fig. 2. The rubber tube was fitted over a glass tube which held the thermometer and through which the thermometer was read. The thermometer was completely immersed in the flowing water and did not touch the walls of the tube except at the top. The small exposed portion of the tube was wrapped with a two-inch layer of cotton. In this was a small curtain which was raised whenever the temperature was observed. Throughout the experiments, except when there was a very slow flow of water through the worm, the temperature of this thermometer was not 0.1° C. higher than a thermometer immersed in the supply tank.

The condensed water was collected in a heavy glass jar and siphoned off to a small reservoir outside in which the water stood at the same level. From this reservoir an overflow connected with large bottles in which the condensed water was stored during an experiment. These bottles were corked immediately at the end of an experiment and kept until measured. The water capacity of this system was found to be quite large and hence its sensitiveness to change in water level was small. From 3 to 5 grams of water could be added in the interior of the calorimeter without producing motion in the system. The sensitiveness of the system was much improved by diminishing the air pressure in the outer reservoir. This was done by an aspirator through a tube, not shown in Fig. 2, connected to the outer reservoir. The relief in pressure was made only at the beginning and end of an experiment. A number of experiments were made by passing known quantities of water into the inner receptacle and measuring the outflow. The aspirator was used for a very short time before water was poured into the vessel and again just as the flow was stopping. The difference in amounts of water put in and drawn off were less than 0.5 grams. This device concedes an error in an actual experiment on account of the extra condensation of from 20 to 30 c.c. of steam drawn over by the aspirator. This error, however, is small in comparison to the error resulting from the water capacity of the system.

The supply tank was about four times the size of the calorimeter and is shown in Fig. 3. It has a constant level attachment and was used to give a constant pressure and hence constant flow to the water passing through the worm. It was surrounded by a constant temperature bath and was made in this special form to give a constant flow of water at 0° C. The incoming water was cooled by ice or a mixture of ice and salt and water at 4° C., collected at the bottom. On being further cooled it rose in the other side of the tank and was drawn off at 0° C. approximately through the worm. The supply tank, however, has not as yet been used in this capacity. Experiments have been made in which the temperature of the water flowing into the worm has had various constant values ranging between 10° C. and 27° C.

The water flowing from the worm was drawn off through a thick heavy rubber tube, curved so that the water condensed on its surface would flow back into the condensed water reservoir. The tube was sealed into the cork steam tight and was forced over a T-tube in the interior of the cork as shown in Fig. 2. A thermometer

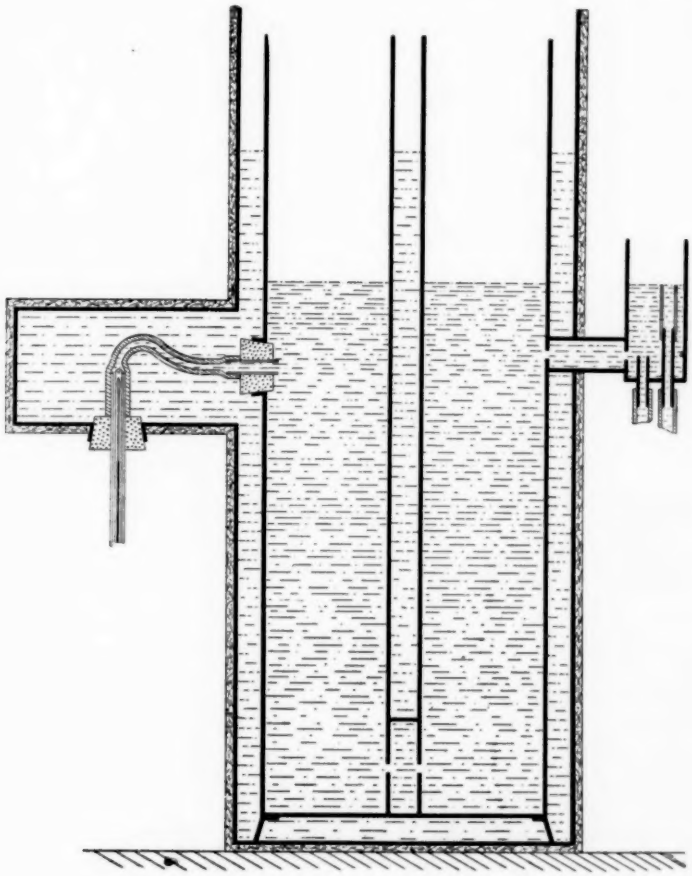

Fig. 3.

was immersed in the water in the tube as shown in the diagram and was kept from touching the sides of the tube by rubber bands at the other end of the tube. The entire T-tube was wrapped with a two-inch layer of cotton and a small curtain removed when the temperature was observed. A small shelf was placed in the cork

in the interior of the calorimeter above this outlet in order to prevent water which was condensed on the sides of the calorimeter from flowing in the condensed water reservoir. While a certain error exists at this outlet to the worm due to conduction of heat through the cork to the water, it is very small on account of the high temperature of the water. It is probably more than counterbalanced, however, by a transfer of heat from the water externally.

Also a certain quantity of heat was lost by conduction externally through the siphon forming the condensed water outlet. This cooling of the siphon and the condensed water contained in it caused an extra condensation of steam on that portion of the siphon inside the calorimeter. The upward bend in the siphon at this point is intended to prevent such condensed water from flowing back into the condensed water reservoir.

The condenser was of a special type as shown in Fig. 2. It was constructed after all other types of condensers used in the laboratory had proved inefficient. In construction it is simply a metal box with a shower of water inside and a pool of water at the bottom with an overflow. The falling water acted as an aspirator and a slow steady stream of air was drawn in through the steam inlet. This prevented any escape of steam into the room. The condenser was capable of condensing all the steam produced by the boiler.

It was thought that the radiation loss as a whole into or from the worm would be large, but this was found to be not so. In the Joly steam calorimeter the radiation loss amounted to 4 or 5 mgm. of condensed water per hour. The radiating surface was small in this calorimeter, but the temperature gradient outward was approximately the same over the entire inner surface of the calorimeter.

In the continuous steam calorimeter, however, the temperature gradient was very steep throughout a large portion of the worm so that in many of the experiments the radiation was internal. The heat was radiated from the walls of the calorimeter into the worm and raised the temperature of the water passing through it about 0.1° C. The radiation correction, however, depended on the values of the temperature of the water flowing in and out of the worm. If this water went into the worm about the temperature of the room and emerged at nearly the temperature of the steam, heat was radi-

ated both to and from the worm and the radiation loss became negligible for some experiments, within the limits of accuracy used in the experiments. However, to measure and correct for radiation, a very thin sheet copper cylinder was used. It almost completely covered the worm, as shown in Fig. 2. It was freely exposed to the action of the steam and rapidly acquired its temperature. A small gutter was constructed around the base of the internal surface of this cylinder and connected with the condensed water reservoir. A similar gutter around the base of the external surface of the cylinder connected with the walls of the calorimeter. The temperature of this copper cylinder was maintained at approximately the temperature of the steam and radiation occurred in both directions, to the walls of the calorimeter and to the worm. The loss of heat from the cylinder to the worm was replenished by condensation on the cylinder and this water was collected in the condensed water reservoir. The condensed water which collected on the cylinder on account of radiation to the walls of the calorimeter was drawn off with the water condensed on the calorimeter walls. There is no radiation under these conditions from the worm outward, not even if the worm acquired the temperature of the steam.

The average temperature of the water going into the worm and that of the water coming out of the worm was obtained in a number of different ways. The thermometers were used to 0.1° C. and were calibrated and standardized before use in ice and steam. No stem corrections were made and errors due to the rising and falling of the mercury column were totally neglected since such a large number of readings were taken under such a variety of conditions. The temperature of the inflowing water never varied as much as 1° C. throughout the total time of the experiment. This change was generally very slow and uniform and readings taken at regular intervals gave a fair average temperature. The variation in the specific heat of water with temperature was totally neglected in considering this average temperature. The temperature of the water flowing from the worm was oscillating within narrow limits. The size and shape of the reservoirs in the worm were varied to produce more uniform mixing of the water before egress. A slow oscillation of about 2° C. in amplitude was generally present in the hot water

thermometer. This oscillation of temperature could be followed absolutely if the thermometer was read at ten or twenty second intervals. A series of experiments on temperature averages carried on conjointly by this method and by an average of readings taken uniformly at three minute intervals, differed only in the fourth place. The temperatures θ_1 and θ_2 were therefore obtained from a series of readings at one, two, or three minute intervals, depending on the quantity of water which was passed through the worm.

The details of operation of a specific experiment were as follows: The boiler was started and the rate of flow through the worm for a definite temperature determined and then made permanent. The apparatus was kept in operation for several hours before beginning an experiment in order to make temperature conditions constant. When ready to begin an experiment, the aspirator was turned on momentarily and the water in the condensed water reservoir reduced to a certain definite level. Immediately the outflow tubes for both m and M were introduced simultaneously into large collecting bottles which possessed only small capillary openings for the escape of air when the bottle was being filled. The temperatures θ_1 and θ_2 were read alternately at regular intervals. Generally from 100 to 250 readings of each temperature were obtained.

At the end of an experiment, the aspirator was again turned on and the water in the internal reservoir reduced to the same level as at the beginning of the experiment. The tubes were removed simultaneously and the bottles immediately sealed. The barometer was read immediately before and after an experiment. The collected water was allowed to cool to the same temperature, generally that of the room. The ratio of the volumes of the water m and M was then obtained by direct measurement instead of the ratio of the masses. The equation

$$L = \frac{v}{V} S(\theta_2 - \theta_1)$$

was used to calculate results. Over 150 experiments were made and the following table consists of eight consecutive experiments taken near the end of the list. The specific heat is taken at unity in the calculations and necessitates a correction for final results.

TABLE I.

θ_1	θ_2	$\theta_2-\theta_1$	v	V	L	P
17.04	93.52	76.48	7212	1027	537.07	761.4
17.71	92.93	75.22	8613	1207	536.76	756.7
21.75	97.15	75.40	14400	2021	537.23	781.4
14.21	91.17	76.96	29987	4293	537.57	770.2
25.16	98.72	73.56	13304	1819	538.01	753.7
15.39	98.21	82.82	11107	1709	538.25	746.1
11.60	90.32	78.72	10241	1497	538.52	741.2
17.73	82.10	64.37	9997	1193	539.40	759.4

When lower temperatures than those shown in the table were taken at the outlet, nearly all the steam was condensed. Radiation into the worm was great and the values of L obtained were very large. The condensed water was cooled below the temperature of condensation of steam and correct values of L could only be obtained by taking the temperature of the condensed water and substituting in equation

$$L = \frac{v}{V} S(\theta_2 - \theta_1) - (100^\circ - \theta_3)S.$$

The values thus obtained did not have the consistency in consecutive experiments that the other experiments possessed. If the results in the table are corrected to the nitrogen scale of temperature and for the variation in the specific heat of water between the required temperature limits, they approximate closely to Callender's standard value of 540.2 calories at 20° C. They average several points lower in value and point to the existence of a small constant error.

The results in the table were obtained directly from prime observations and compared very favorably with the best results by other methods, even though their measurements be made with the greatest degree of accuracy, and have all the necessary corrections. Thus Joly with the steam calorimeter in six experiments on the specific heat of water between 10° C. and 100° C. obtained results in which the measurements varied from 1.0044 to 1.0087. Barnes with the continuous electrical method obtained results in five experiments on the value of J, the mechanical equivalent of heat, at 29° C.,

which vary between the limits 4.1771 and 4.1803. These two methods are probably the most accurate heat methods known. The method of mixtures, with the greatest possible refinement in the measurements, may acquire the same degree of accuracy, but the corrections are too large and probably large constant errors exist. The continuous method of steam calorimetry as here outlined, is capable of a much greater refinement. The small constant error is probably due to the existence of air in the water and the subsequent production of bubbles in the outflowing water (m). The quantity of air is small and the heat absorbed in raising its temperature may be neglected. The existence of water vapor in these bubbles and the consequent loss of heat on account of the high latent heat of vaporization cannot be ignored. It probably fully accounts for the lower average value of the latent heat which was obtained. It probably also accounts for a portion of the difference in the values obtained, since its effect was irregular and depending on the temperature of the outflowing water.

An attempt to eliminate this source of error was only partially successful. The pressure on the water was raised, by raising the level of the outflow five or six feet and resulted in a diminution of the number and size of the bubbles. A number of experiments have been made with special care which have given concordant results in consecutive experiments which differ only in the fifth significant figure. The constant errors enter to a much greater extent and have not been as yet completely classified. An absolute determination of the latent heat of condensation of steam is being made with the greatest refinement and this paper is a preliminary one and intended simply as a development of the method.

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ON THE DIFFERENTIAL TELEPHONE.[1]

By William Duane and Charles A. Lory.

1. The authors have tested the merits of a "differential telephone" as a means of measuring a self-inductance, and have devised an exceedingly accurate method of measurement. The differential principle has been used before in connection with a telephone,[2] but the accuracy claimed for it in the measurement of a self-inductance was only about one per cent. With the instrument described below an accuracy greater than one fiftieth of one per cent. has been attained.

2. A differential telephone receiver is one on the bobin (or bobins) of which two coils AA' and BB' (Fig. 1) are wound side by side. By suitable means these coils are adjusted so as to have equal self-inductances and equal resistances, and so that the magnetizing effect of a current flowing through one coil from A to A' is exactly annulled by that of an equal current flowing through the other coil from B to B'. A variable self-inductance standard S is joined in series with coil AA', and the coil of unknown self-inductance X in series with coil BB'. A non-inductive variable resistance R is inserted in series with S or X according as the

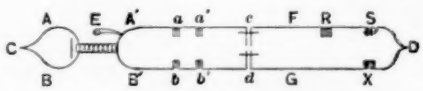

Fig. 1.

resistance of X is greater or less than that of S. The two entire circuits are then joined in parallel and an alternating electromotive force is applied to the branch points C and D. By varying R and S values can be found easily such that no sound is heard in the receiver. When this is the case the two parallel currents must be

[1] Presented at the St. Louis meeting of the American Association for the Advancement of Science, December, 1903.

[2] H. Ho, Electrical World and Engineer, May, 1903.

equal to, and in phase with each other, and therefore the self-inductance of X must equal that of S.

3. Before the instrument is ready for use, three adjustments are necessary, but these may be made once for all, and do not have to be repeated before each measurement.

First, the magnetic effects of the two receiver coils must be equalized. This is the most difficult adjustment, as it is impossible to wind the two coils exactly alike. The equality of the magnetic effects can be tested by joining the ends A' and B together, and sending an alternating current through the two coils in series in the direction $AA'BB'$. The same current then passes through each coil in the proper direction, and the magnetic effects are equal and opposed to each other, if no sound is heard in the receiver. In making the receiver it is best to wind the two coils side by each and as close together as possible, and each layer of turns should be evenly distributed. If in the first layer the turns belonging to coil AA' are nearer the diaphragm than those belonging to coil BB', in the second layer the positions of the turns belonging to the respective coils should be reversed, and so on alternately in the succeeding layers. Even with this careful winding the magnetic effects are not exactly equal to each other; in our instrument the difference between them amounted to the effect of a fraction of a single turn. A number of schemes for balancing the residual effect were tried, but the only one that completely extinguished the sound in the receiver was the insertion of a small coil E of about ten turns of wire in series with one of the receiver coils. E was mounted on the frame of the receiver in such a way that it could be shifted about with its plane always parallel to the common axis of the coils AA' and BB'. A position could be found for E that completely extinguished the sound, even though an alternating electromotive force of twenty volts was applied to the coils.

The second and third adjustments are the equalizations of the resistances and self-inductances of the coils. The self-inductances can be equalized by placing in series with each receiver coil a pair of coils (aa' and bb'), and by altering the distances between the coils in each pair. The resistances can be equalized by means of sliding contacts moving along pairs of parallel wires (c and d). The

test for the equality of the resistances and of the self-inductances is accomplished by joining the circuits from A to F and from B to G in parallel, applying an alternating electromotive force to the branch points and varying the adjustments until no sound is heard in the receiver. After these adjustments have been made the instrument is ready for measurements.

4. In comparing the self-inductance of a coil X with that of a standard S two balances must be made. The resistance of the parallel circuits must be made equal to each other by varying R, and the self-inductances equal to each other by varying S. One of these balances does not depend upon the other (as is the case, for instance, in Maxwell's method of comparing a self-inductance with a capacity), and hence the double balance is much easier than it otherwise would be.

If the divisions of the resistance box R are not small enough, the final balance of the resistances may be made by means of the sliding contacts c and d.

Absolute silence in the telephone receiver can be obtained easily, if there is no iron in the self-inductance. With an iron core a very sharp minimum can be obtained, but not in general total silence. If, for instance, the coil X is a Morse telegraph sounder, several tones of different pitches can be heard, and values of S and R can be found that extinguish the most prominent tone.

5. The accuracy of the measurement may be estimated by means of the following calculation. Suppose that the resistance of the two parallel circuits are equal to each other and each equal to r, and further that the reactances of the two circuits differ from each other by the small amount dx, x being the reactance of one circuit and $x + dx$ that of the other. The complex expressions for the impedances of the two circuits are ($j = \sqrt{-1}$).

$$Z_1 = r + jx, \qquad \text{and} \qquad Z_2 = r + j(x + dx).$$

If e is the simple alternating electromotive force applied to the branch points C and D, and I_1 and I_2 are the currents in the two branches respectively,

$$I_1 = \frac{e}{r + jx}, \qquad \text{and} \qquad I_2 = \frac{e}{r + j(x + dx)}.$$

The sound heard in the receiver depends upon the difference between the two currents, and, if dI is this difference,

$$dI = I_1 - I_2 = e\left[\frac{1}{r + jx} - \frac{1}{r + j(x + dx)}\right].$$

Reducing to a common denominator, neglecting small quantities of higher orders than dx and eliminating the imaginary unit j, we find that the absolute magnitude of dI is

$$dI = \frac{e\,dx}{r^2 + x^2}.$$

If dI is the smallest current that can be detected, when it flows through one of the receiver's coils, dx cannot be greater than

$$dx = (r^2 + x^2)\frac{dI}{e}.$$

To estimate the numerical value of the accuracy assume that the resistances r and the reactance x are each 50 ohms, that e is one volt and that a current of one millionth of an ampere can be detected by the telephone. These values are not unusual. Then

$$dx = \tfrac{1}{2000} \text{ ohm},$$

and the percentage error is

$$100\frac{dx}{x} = \tfrac{1}{1000} \text{ per cent.}$$

The reactance x may be taken to be that of the standard S alone, for the reactance of the rest of the branch circuit is very small. The reactance is due to self-inductance, and therefore the percentage error in the measurement of the self-inductance is also $\frac{1}{1000}$ per cent.

As a matter of fact the accuracy does not fall far below the theoretical value. It is not difficult to measure a self-inductance to within one twentieth of one per cent., and with care the accuracy can be considerably increased.

6. Electrolytic resistances also may be measured by means of the differential telephone. For this purpose the coil X is replaced by a vessel with suitable electrodes and containing the electrolyte, and the standard S is removed.

7. The advantages of using a differential telephone as described above for measuring a self-inductance are: (*a*) that the apparatus is portable and does not get out of order easily; (*b*) that great accuracy can be attained and the manipulation is not difficult; (*c*) that the heating of the circuits by the currents does not alter the self-inductance balance, and therefore that large electromotive forces may be used; and (*d*) that only one standard is required; it is not necessary to know the values of any resistances or the lengths of a bridge wire. The disadvantage is that the self-inductance of the standard must equal that of the coil to be measured. If, however, the self-inductance to be measured is smaller than the smallest inductance obtainable in the standard, a fixed standard may be placed in series with the unknown coil, and the sum of their self-inductances measured; and if the self-inductance to be measured is greater than the greatest inductance obtainable in the standard, a fixed standard may be placed in series with the variable standard. Thus the range of measurement may be increased indefinitely beyond the limits of the variable standard.

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ON THE DETERMINATION OF REFRACTIVE INDICES BY MEANS OF CHANNELED SPECTRA.

BY S. R. WILLIAMS.

IF the light reflected from two parallel optical surfaces be examined with a spectroscope of sufficient resolving power, the resulting spectrum will be traversed by dark bands corresponding to those wave-lengths which interfere, the center of a dark band occurring when $2\mu D = 2N\lambda/2$ and the medium between the surfaces is of a less refractive index than that composing the optical surfaces. For the center of a light band $2\mu D = (2N+1)\lambda/2$, where D is the linear thickness of the space intervening, λ the wave-length and N some intregal number. In case μ becomes equal to unity, as occurs when we have a vacuum, the value of N in the above equation may be derived, giving the length of the path D in wave-lengths of light. Using the sun as a source of light there will appear in the spectrum the Fraunhofer lines, in addition to the interference bands, thus furnishing a scale of reference for every interference band.

If two dark bands appear in determinate parts of the spectrum corresponding to wave-lengths λ_1 and λ_2, then by the above equations $2\mu_1 D = N\lambda_1$ for the longer wave-lengths, and $2\mu_2 D = (N+m)\lambda_2$ for the shorter, whence assuming $\mu = 1$ we obtain

$$N = \frac{m\lambda_2}{\lambda_1 - \lambda_2}.$$

A comparison of the results obtained by various investigators in the determination of refractive indices shows at once that the optical thickness cannot be obtained by the above formulæ for those substances in which μ is such an uncertain quantity.

Dr. D. B. Brace[1] has shown that the difference in path of two

[1] Phil. Mag. (48), 1899, p. 345; (1), 1901, p. 339.

interfering rays, or the optical thickness, may be determined independently of the factor μ. His method is to take a cell, whose thickness can be varied, (usually a wedge-shaped cell) and if, in the variation of the thickness, m bands are shifted past a definite point in the spectrum for which the difference in path N is to be determined, and n is the ratio of the bands between two definite wavelengths, λ_1 and λ_2, before and after the variation in thickness, then

$$N = \frac{mn}{1 - n}.$$

Being able to determine the difference in path thus, we are furnished with the means for determining the value of μ for any transparent substance. For, if we take as a medium of reference a substance of index μ_r, which is accurately known, $2\mu_r D = N_r\lambda$. In the body whose index is to be determined, $2\mu_e D = N_e\lambda$. D and λ being made the same, we have

$$\mu_e = \frac{N_e}{N_r}\mu_r,$$

that is, the refractive index of any substance is equal to the ratio of the optical lengths of equivalent paths in that substance, and the medium taken as a reference, multiplied by the index of the reference film.

For liquids the path D may be easily made the same by making a thin wedge-shaped cell with its sides parallel, and allowing the light to be reflected from a narrow vertical section of the same. If the upper half of the narrow section reflects from the surfaces of the film taken as a reference, and the lower half from a substance whose index is to be determined, there will occur in the spectrum, simultaneously, two sets of interference bands from which the orders of interference may be calculated, and from these the index.

By the distribution of the two sets of bands thus obtained, the dispersive power of the two substances may be compared, for if μ was larger in one than in the other, the bands would appear more frequent throughout the spectrum as shown by our equation above. In this way the complete record of dispersion of a body may be obtained throughout the spectrum. Furthermore the bands superimposed upon the Fraunhofer lines as a scale may be photographed,

and a permanent record made of the dispersion of the substance examined.

To make this a practical working method was the object of this investigation.

A four-inch Rowland concave grating, 14,438 lines to the inch, with Brashear mounting, and focal length of ten feet was used. Fig. 1 shows the arrangement of the optical system. The slit was illuminated by the direct rays of the sun, and an image of the same was formed on the wedge W by the lens L. In the usual mounting of the Rowland concave grating, the slit or its image should be

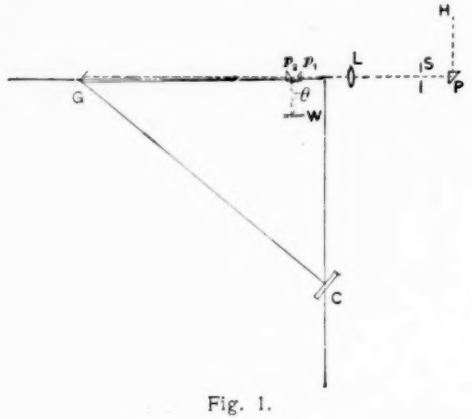

Fig. 1.

placed over the intersection of the ways, in order that the spectrum may be brought to a focus on the proper focal curve. Instead of doing this the light coming from the slit S through the lens L was reflected by means of the prism p_1 upon the wedge W, where the image of the slit was formed. The two surfaces of W returned the light to the prism p_2, which reflected it upon the grating G. The distance of W from p_1 and p_2 was such, that the image in p_2 was at the intersection of the ways, and thus the conditions for accurate focusing were secured. The image of the slit at W served two purposes, in that it brought the Franuhofer lines and interference bands into focus at the same time, and also afforded the means of getting a narrow vertical section of the cell.

Plate glass cut in strips, 3 x 13 cm., and 6 mm. thick, formed the sides of the wedge-shaped cell. To keep the plates apart and also

form the bottom and ends of the cell as well as the dividing strip between the films, cork strips about .75 mm. thick were cut out, as shown in Fig. 2. The plates were then held firmly against this strip by means of clamps, those on one end being screwed down more tightly than the other, in order to make the cell wedge-shaped. The mounting of the cell was so arranged by means of a rack and pinion, that it could be raised and lowered in a vertical direction, and also moved in the direction of its length, by means of a micrometer screw. For photographing the two sets of bands simultaneously the cell was so moved that the image of the slit occupied a position as *ab* in Fig. 2.

ABCD is that part of the cell containing the film of reference, while the rest of the cell holds the liquid whose index is to be determined. The two sets of bands having been photographed together the cell was moved parallel to its length, while the bands

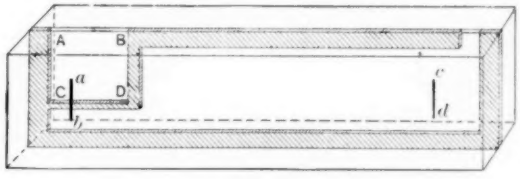

Fig. 2.

from the liquid film were counted as they moved past some fixed point in the spectrum, until the image of the slit occupied a position as *cd* in Fig. 2. In this position a set of bands throughout the spectrum was photographed, and knowing the index for one wavelength, it could be determined for all others by counting the bands.

Air was used for a reference film. Its index has been very accurately determined, by means of the interferometer, and the presence of a vapor does not effect it, as it has been ascertained that the refractive index of dry air is only about one millionth greater than air saturated with a vapor. The values as found by various investigators, vary by little over one part in a million. Air has also been used as a medium of reference by other investigators, and so is used in this work.

The direction of variation in thickness of the cell is at right angles to the interference bands, since along their length, the thick-

ness must be uniform. They will be parallel to the Fraunhofer lines then, when the refracting edge of the cell is parallel to the image of the slit. This was the means used for getting the plates of the cell, parallel in a vertical direction, for, if the interference bands were brought into coincidence above and below the dividing strip, before the liquid is introduced, the value of D for both films must be the same.

Another condition necessary, to give D an equivalent path, in both films, is, that the incidence be normal upon their surfaces, for, owing to refraction at the bounding surfaces the ray will be bent more or less for one than the other depending upon the difference in index, consequently D would vary for the two films. In the system used, normal incidence was not possible, but it was made so small, that it entered no appreciable error. Let AO, Fig. 3, represent the path of normal incidence upon air film xy

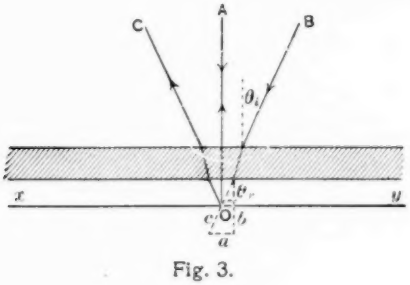

Fig. 3.

contained between two plates. Let BOC be the path of a ray at an incident angle of θ_i. Since the film xy is an air film, the angle of refraction at its surface will be the same as the angle of incidence on the plate ($\theta_i = \theta_r$). θ_r will also be a maximum for air, hence if it can be shown that the angle θ_r does not produce an appreciable error in difference between c and b, which are the lengths of D at normal and oblique incidence, then it can be shown that for a ray incident at an angle θ_i upon two films, as an air and a liquid film, the difference in D for the two is not an appreciable error. For the difference in the angle of refraction at the surfaces of the two films, can never be as large as θ_r, consequently the difference in length can never be as great as $c - b$ in the figure.

$$\tan \theta_r = \frac{a}{b}$$

$$\sin \theta_r = \frac{a}{c}$$

$$c - b = a\left(\frac{1}{\sin \theta_r} - \frac{1}{\tan \theta_r}\right)$$

$$\frac{c-b}{b} = \tan \theta_r \left(\frac{1}{\sin \theta_r} - \frac{1}{\tan \theta_r}\right) = \sec \theta_r - 1$$

but $(c - b)/b$ is the per cent. of error introduced. In the apparatus as set up, the light at prism P_1, Fig. 1, was diaphragmed so that θ_r was about 25′. Substituting this value in the above equation, shows an error of about three parts in a hundred thousand. With light incident on two films the error due to difference of path D is still less, as shown above.

In the liquid films with oblique incidence, dispersion produces a variation in D for the different wave-lengths, so that if the index of the substance under examination be smaller than that of the containing cells, the path D for the violet ray, will be greater than that of the red, since it will be bent away from the normal more than the less refrangible wave-lengths. We have shown that difference in path, due to refraction, produced a negligible error, and since the angle of deviation is always less than the angle of refraction, no error in difference of D for the various wave frequencies will occur.

Again the length of D, as determined from the bands in the photographs, will be augmented by change of phase, at reflection from the bounding surfaces of the film, if the conditions of reflection for the two beams are not the same. In order to increase the intensity of light in reflection from the surfaces, a full silvered surface was used at the back. After repeated trials, it was found that an approximate half silvered front surface, which gave a brighter image of a flame by reflection, than by transmission, product the blackest bands in the channeled spectra.

O. Wiener[1] has found that the change of phase by reflection, varies with the thickness of the silvering, increasing as a retardation from 0 to $3/4\lambda$, for all wave-lengths, as the thickness of silver increased from zero to a full silver.

[1] Wied. Annal. (31), 1887, p. 647.

The index of silver is about .25, so that in the cell that was used, if the half silvering was placed on the rear surface of the front plate, and the full silvering on the front surface of the rear plate, then for both films, the condition of reflecting from a less refracting substance to a greater, was secured for the two interfering rays.

According to Wiener, the half-silvered surface introduced a retardation of about $\lambda/2$, while at the rear surface it was $3\lambda/4$. This made the difference in phase of the two interfering rays, greater by about $\lambda/4$, since the ray reflected from the back surface has been retarded that much more than the one reflected from the front surface. With the order of interference that was used in this work, this entered an error of less than three parts in one hundred thousand in determining the order.

In making the computation of indices, this change of phase was not regarded, and since in photographing, the bands from the two films were brought into coincidence with the D_1 line, the orders of interference for these were regarded as an odd multiple of $\lambda_D/2$.

$$(2N + 1)\frac{\lambda}{2} = 2\mu D$$

then becomes the formulæ for finding the order of an interference band.

The cell was housed in a pasteboard box, which kept the temperature uniform during the photographing, so that the bands did not shift due to that cause. The temperature was taken at the beginning of the series of photographs, and is indicated on the plates.

At times, a shift of the bands did occur, but this was due to a yielding of the cork strip between the plates, or other mechanical effects. It was found that if the cell was allowed to stand before using, no difficulty was encountered from this source, for an eye piece was attached to the side of the camera box, whose focus was on the focal curve of the grating, and through this the bands were watched to see that they did not shift with respect to the Fraunhofer lines.

Mitchell has shown that the astigmatism of a grating depends upon the length of the illuminated portion of the slit, the length of

the ruled lines, and the angle which the incident light makes with the axis of the grating. On account of this property, comparison work is not possible as in the prismatic spectrum. In photographing the two sets of bands, comparison work was done, but other conditions were introduced that do not occur in ordinary comparison work. The light falling upon a grating was slightly divergent, and the dividing strip in the cell, cut out a narrow section of light across the middle of the ruled portion of the grating. The light above this dark strip came from one film, and that below from the other. The length of ruling that was producing the spectrum for one set of bands, was therefore less than half the actual length of ruling of the grating, and as a result the interference bands, from one film, extended about half the width of the spectrum.

If D is the thickness, μ the index, λ the wave-length, and N is the order of interference, the distribution of the bands throughout the spectrum is represented by the equations:

(1) $$N = \frac{2\mu D}{\lambda}$$

(2) $$N = \frac{2D}{\lambda}$$ where $\mu = 1$, as for a vacuum.

Differentiating these we have,

(3) $$\frac{dN}{-d\lambda} = \frac{2D}{\lambda^2}\left(\mu + \lambda\frac{d\mu}{-d\lambda}\right),$$

(4) $$\frac{dN}{-d\lambda} = \frac{2D}{\lambda^2}.$$

Equations (1) and (2) are the equations of an equilateral hyperbola, N and λ being the variable parameters. In (3) and (4) the left hand side of the equation, represents the number of bands between wave-lengths λ and $\lambda - \delta\lambda$, and is expressed on the right hand side in terms of D, λ and μ. The variation of the order with wave-lengths in the two equations, (3) and (4), differ by the factor in parenthesis which expresses the difference in variation between the two sets of bands. Since in the two equations $2D/\lambda^2$ appears, which is the expression for the variation of order due to wave-length alone, if we superimpose a set of bands, *e. g.*, those in a vacuum,

[1] J. Hopkins, Univ. Circulars, June, 1898.

whose variation is represented by (4), upon a set of bands represented by (3), the differential effect will be that due to the dispersion of the substance whose bands are represented by (3). To do this, the bands from the two films are brought into coincidence, say in the less refrangible portion of the spectrum. If now we go toward the violet, the bands in the spectrum whose variation is represented by (3) will, in general, increase in number more rapidly than those used as a scale. Consequently they will get out of coincidence. If then they should come into coincidence again, due to the bands not increasing so rapidly as those on the scale, the term in parenthesis must diminish, which indicates anomalous dis, persion. In a subsequent paper, it will be shown more in detail how this method may be used for the detection of this characteristic.

Preliminary work has been done in adapting this method to the determination of refractive indices of solids. The optical system was, in general, the same as used in working with liquids. The difficulty with solid wedges, is to get a reflecting surface for the wedge and an equivalent film of air. If silvered plates be placed on both sides of the solid wedge, as for the liquid film, it is impossible to get the plates in close enough contact to the wedge, so as not to have thin air films intervening. This difficulty was overcome, by measuring these air films, and thereby obtaining the equivalent film of air for reference. In Fig. 4 is shown a vertical cross section of the cell used in getting the order of interference in the wedge and an equivalent film of air. W is the solid wedge between two glass plates, silvered as they were in the liquid cell described above. The path of the light at A gives the total thickness of air space, B that of the wedge and air films on both sides. By properly silvering the lower part of the wedge, the interference bands due to the wedge itself, may be obtained in C. $B - C$ gives the thickness of the air films on the two sides of the wedge. $A - (B - C)$ gives the thickness of an air film equivalent to the wedge. Consequently $C/[A - (B - C)]$ gives the index as compared with air. It was impossible to get the three sets of bands simultaneously, but A and B were first obtained and photographed, and then by means of rack and pinion, the cell was raised and B and C obtained.

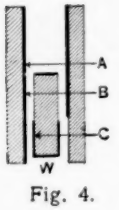

Fig. 4.

With proper adjustments, there was no shift of the bands in raising the cell vertically, since its sides and that of the wedge are parallel in that direction.

In solids showing double refraction, the optical system was arranged as shown in Fig. 5, with a nicol, A, before the slit and another one, E, between the prism p_2 and the grating.

If a ray of light, polarized at an angle of 45° to the principal axis of a crystalline wedge, pass normally through and is reflected back again, the emergent light, when resolved by a spectroscope,

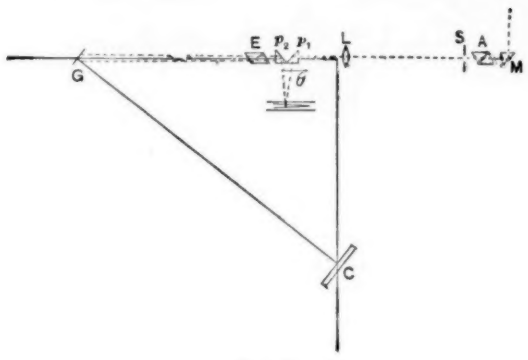

Fig. 5.

will be traversed by interference bands due to double refraction in addition to the ordinary bands arising from the wedge. The superposition of the two sets of these latter bands, give rise to a maximum and a minimum in the intensity, producing the bands from double refraction referred to. The formulæ for these minima, when the nicols are crossed, is $N = 2D[(\mu_e - \mu_0)]/\lambda$. In this way both the bands due to interference between the beams reflected between the front and back surface, and those due to differential double refraction may be photographed.

With double refracting crystals, the system of reflection at p_1 and p_2 had to be changed, and is shown in Fig. 6. In using the prisms p_1 and p_2 only, there would be a change of phase between the component vibrations. By using a double system of reflection, at right angles to each other, as shown in the cut, the two components will have practically the same change of phase introduced. This double reflection will place the image of the slit in a horizontal

position, which necessitates turning the wedge so that its length is at right angles to the image of the slit.

Plate I. is a series of photographs for triple-distilled water. The photographs are arranged with the shorter wave-lengths to the left. In each set of photographs, No. 4 was the first setting made in the spectrum, and is that of the liquid and air bands together, between the Fraunhofer lines, β_2 and D_1. No. 3 was taken in the same portion of the spectrum, after shifting several hundred bands. The others were taken in the order 5, 6, 2, 1.

In Plate I. the number of air bands between β_2 and D_1 is 276.6 from which the order 1978.5 for the Fraunhofer line D_1 is obtained by the formulæ

$$N = \frac{m\lambda_2}{\lambda_1 - \lambda_2}.$$

In the liquid bands there are 374.7 between these two wave-lengths, and after shifting 338 bands past the D_1 line in the spectrum and photographing again, there are by count 326.7 bands in this same region, substituting these values in our equation

$$N = \frac{mn}{1 - n} = \frac{338 \times \frac{326.7}{374.7}}{\frac{48}{374.7}} = 2300.5.$$

The order as found for the liquid is that of the D_1 line in photograph No. 3. For the D_1 line in No. 4 we add 338 to 2300.5 = 2638.5. The ratio between these two orders,

$$\frac{N \text{ liquid}}{N \text{ air}} = \mu = \frac{2638.5}{1978.5} = 1.3336.$$

For the D_1 line in No. 3, the order for air would be the order for liquid at that point, divided by the index,

$$\frac{2300.5}{1.3336} = 1725.0.$$

From these two orders of air and liquid for the D_1 line, the orders for all other points in the spectrum may be calculated and counted from the photographs.

In counting the bands, they were laid on a dividing engine under a micrometer eye piece, and as the photographs were shifted under the cross wires, the bands were counted. The fractional part of a band being estimated only to tenths.

The results from all of the photographs shown, were obtained in the manner described above, and are tabulated below.

PLATE I.

Triple-distilled Water. $m = 338$. *Temp.* 22°.

λ cm.	N, Air.	N, Liquid.	Index.
6.8674×10^{-5}	1480.9	1971.1	1.3310
6.5630 " "	1549.9	2063.8	1.3316
5.8961 " "	1725.0	2300.5	1.3336
5.6169 " "	1810.7	2416.9	1.3348
5.3283 " "	1908.7	2549.2	1.3356
5.2270 " "	1945.8	2599.4	1.3359
5.1728 " "	1966.2	2627.4	1.3363
4.9577 " "	2051.4	2743.8	1.3375
4.8615 " "	2092.1	2799.4	1.3381
4.6680 " "	2178.7	2917.6	1.3391
4.3837 " "	2320.1	3111.0	1.3409
4.2269 " "	2406.2	3228.5	1.3417
4.1019 " "	2479.5	3330.5	1.3432
3.9686 " "	2562.6	3445.0	1.3443

PLATE II.

Absolute Alcohol. $m = 393$. *Temp.* 22°.

λ cm.	N, Air.	N, Liquid.	Index.
6.8674×10^{-5}	1268.0	1721.0	1.3572
6.5630 " "	1326.4	1802.1	1.3586
5.8961 " "	1476.2	2009.5	1.3612
5.6169 " "	1549.5	2111.5	1.3627
5.3283 " "	1633.4	2227.5	1.3637
5.2270 " "	1665.1	2271.5	1.3642
5.1728 " "	1682.5	2296.0	1.3646
4.9577 " "	1755.5	2397.7	1.3655
4.8615 " "	1790.3	2446.3	1.3664
4.6680 " "	1864.4	2550.4	1.3679
4.3837 " "	1985.5	2721.3	1.3705
4.2269 " "	2059.1	2825.1	1.3720
4.1019 " "	2121.9	2913.5	1.3731
3.9686 " "	2193.0	3014.5	1.3746

PLATE III.

α-Monobromonapthalin. $m = 475$. *Temp.* 24°.

λ cm.	N, Air.	N, Liquid.	Index.
6.8674×10^{-5}	1440.1	2374.0	1.6485
6.5630 " "	1507.2	2489.2	1.6515
5.8961 " "	1677.5	2787.5	1.6617
5.6169 " "	1760.9	2934.6	1.6665
5.3283 " "	1856.2	3103.6	1.6720
5.2270 " "	1892.2	3168.2	1.6743
5.1728 " "	1911.0	3204.1	1.6767
4.9577 " "	1994.9	3354.9	1.6817
4.8615 " "	2034.5	3427.5	1.6847
4.6680 " "	2118.7	3584.6	1.6918
4.3837 " "	2256.2	3844.7	1.7034
4.2269 " "	2339.5	4007.6	1.7130
4.1019 " "	2411.2	4149.7	1.7210
3.9686 " "	2492.1	4322.5	1.7345

One of the difficulties met with in this method, is the shifting of the bands during the photographing as occurs in all interference work. This was mechanical, and was overcome by shifting the cell, for all the shift of the bands, with respect to the Fraunhofer lines, originated in the cell itself. That the Fraunhofer lines and

PLATE IV.

Oil of Cinnamon (Cassia). $m = 273$. *Temp.* 25.5°.

λ cm.	N, Air.	N, Liquid.	Index.
6.8674×10^{-5}	1229.0	1919.1	1.5615
6.5630 " "	1286.3	2014.2	1.5659
5.8961 " "	1431.6	2257.5	1.5769
5.6169 " "	1502.7	2379.3	1.5833
5.3283 " "	1584.0	2519.0	1.5903
5.2270 " "	1614.8	2572.5	1.5937
5.1728 " "	1631.7	2602.4	1.5949
4.9577 " "	1702.4	2728.0	1.6024
4.8615 " "	1736.2	2788.5	1.6061
4.6680 " "	1808.1	2920.0	1.6149
4.3837 " "	1925.5	3141.0	1.6312
4.2269 " "	1997.0	3283.5	1.6437
4.1019 " "	Absorption, no bands visible.		
3.9686 " "	" " " "		

Plate V.

Benzole. $m = 235$. *Temp.* 22°.

λ cm.	N, Air.	N, Liquid.	Index.
6.8674×10^{-5}	1298.7	1924.9	1.4822
6.5630 " "	1359.2	2017.7	1.4844
5.8961 " "	1512.8	2254.4	1.4902
5.6169 " "	1588.0	2371.6	1.4934
5.3283 " "	1673.9	2505.4	1.4967
5.2270 " "	1706.4	2556.4	1.4981
5.1728 " "	1724.3	2584.4	1.4988
4.9577 " "	1799.0	2702.5	1.5022
4.8615 " "	1834.7	2758.9	1.5037
4.6680 " "	1910.6	2880.4	1.5076
4.3837 " "	2034.7	3079.4	1.5134
4.2269 " "	2110.2	3202.8	1.5177
4.1019 " "	2174.5	3308.4	1.5214
3.9686 " "	2247.4	3429.4	1.5259

interference bands appear simultaneously, is one of the important features of this method. No matter what apparatus is used, the scale of the Fraunhofer lines will always be the same, from which the measurements are made, and no mechanical errors can enter. The shift of bands which might arise due to changes of temperature, must be guarded against in all methods.

Plate VI.

Olive Oil. $m = 267$. *Temp.* 26°.

λ cm.	N, Air.	N, Liquid.	Index.
6.8674×10^{-5}	1190.3	1721.0	1.4458
6.5630 " "	1245.7	1802.1	1.4474
5.8961 " "	1386.5	2012.5	1.4514
5.6169 " "	1455.4	2116.5	1.4542
5.3283 " "	1534.2	2234.5	1.4565
5.2270 " "	1564.0	2279.5	1.4567
5.1728 " "	1580.3	2304.2	1.4581
4.9577 " "	1648.8	2407.6	1.4602
4.8615 " "	1681.5	2457.5	1.4615
4.6680 " "	1751.1	2563.5	1.4639
4.3837 " "	1864.9	2733.0	1.4655
4.2269 " "	1934.0	2839.5	1.4682
4.1019 " "	1992.9	2930.6	1.4705
3.9686 " "	2059.8	3034.5	1.4732

It is also possible with the normal spectrum, to determine directly the index for any wave-length. In the prism method, we are dependent upon the more prominent Fraunhofer lines, for points of reference.

In conclusion, the author desires to express his thanks to Professor Brace for valuable assistance rendered him, whereby he was enabled to carry out this research.

PHYSICAL LABORATORY,
UNIVERSITY OF NEBRASKA, June, 1903.

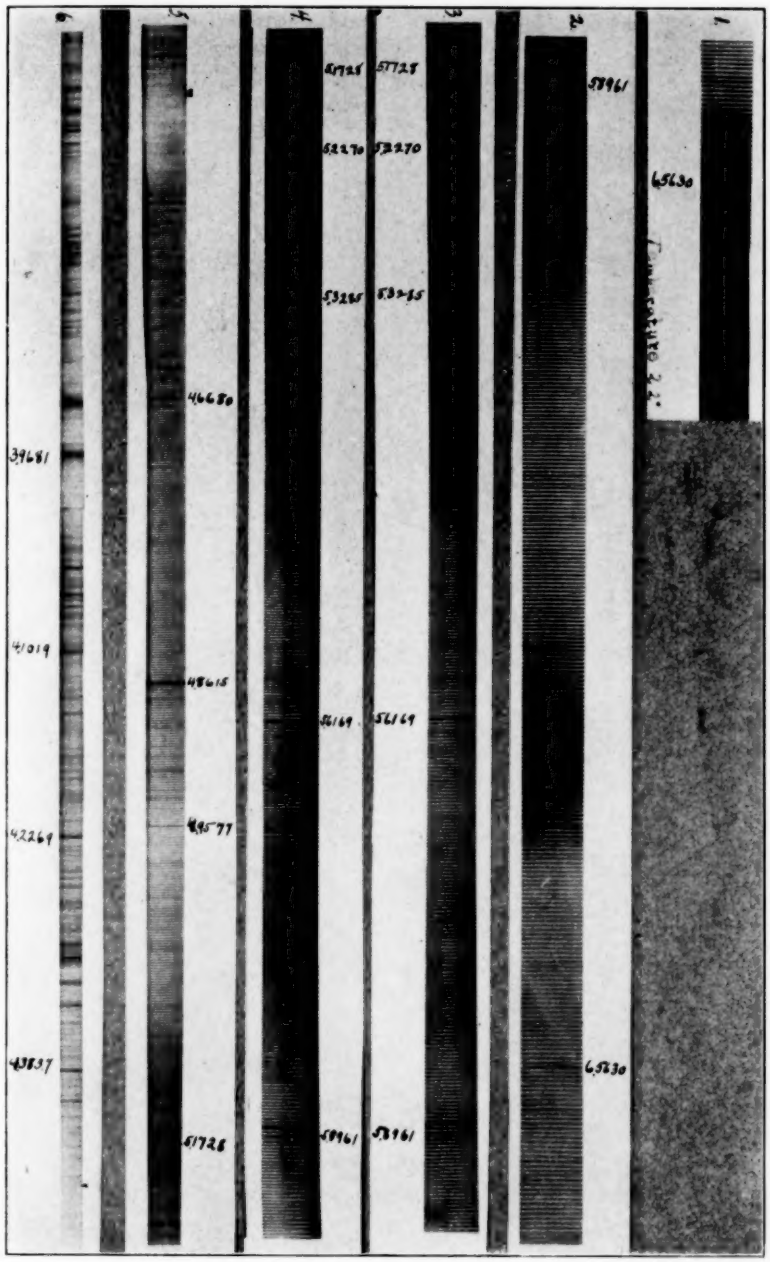

PLATE I.

S. R. WILLIAMS

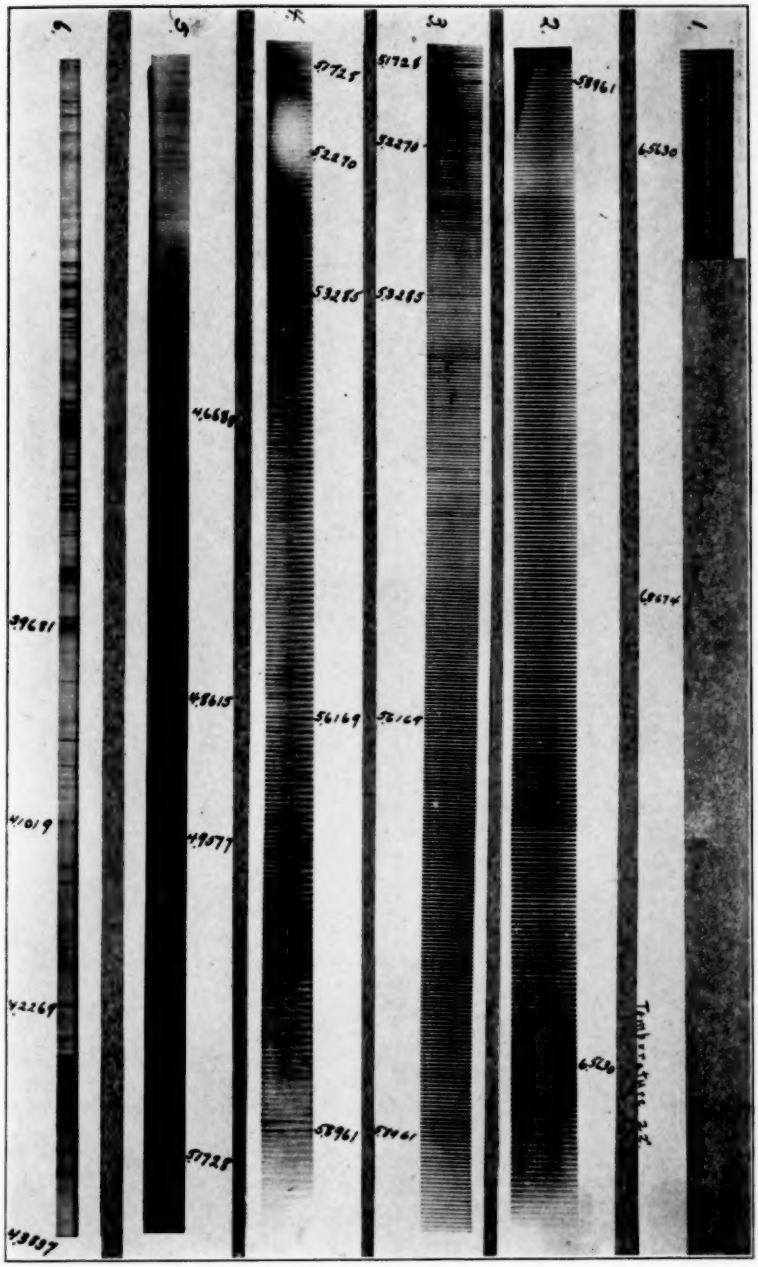

PLATE II.
S. R. WILLIAMS.

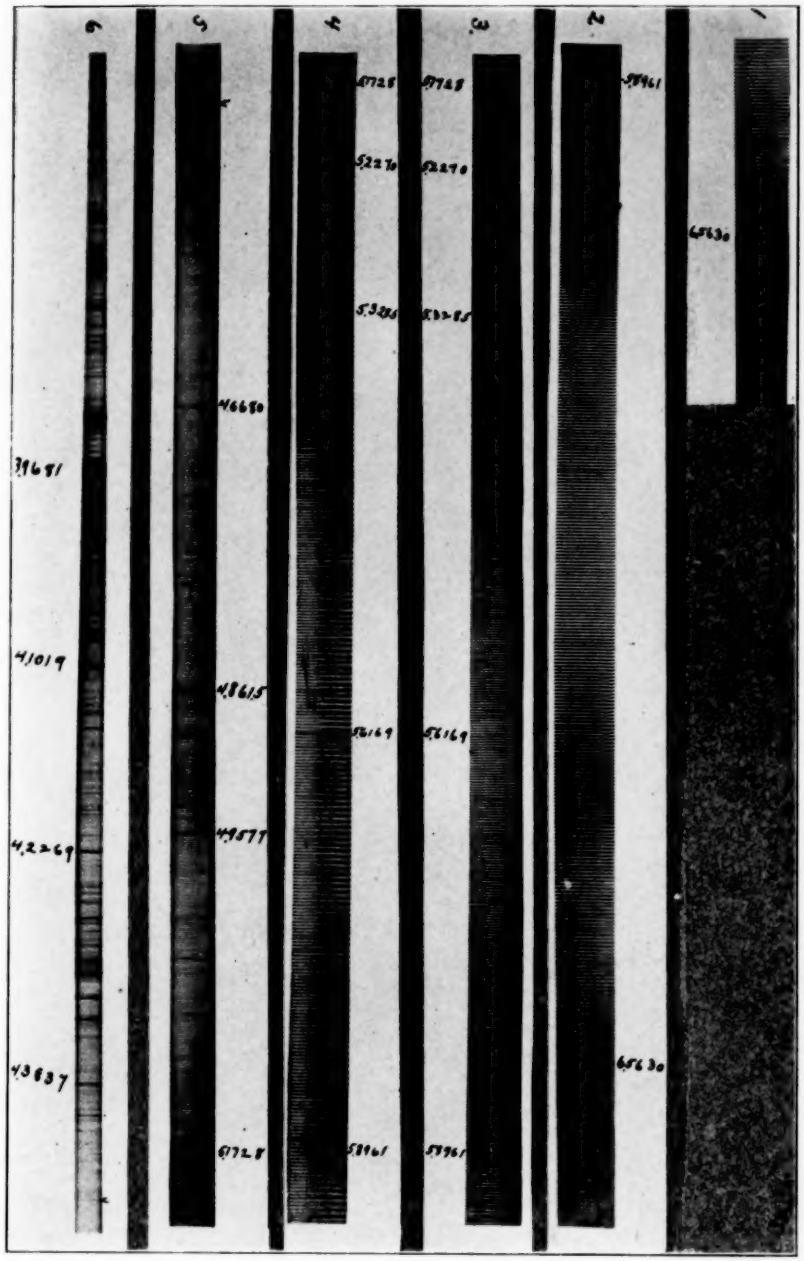

PLATE III.
S. R. WILLIAMS.

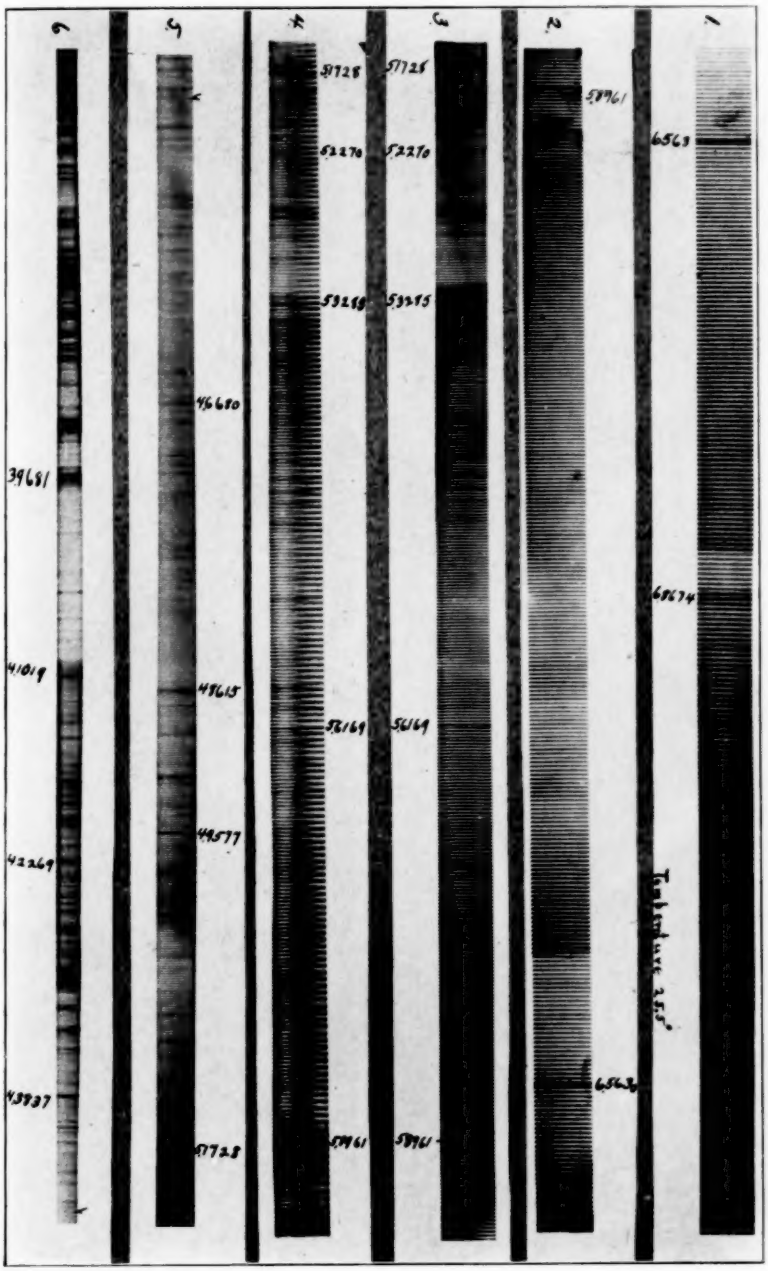

PLATE IV.

S. R. WILLIAMS.

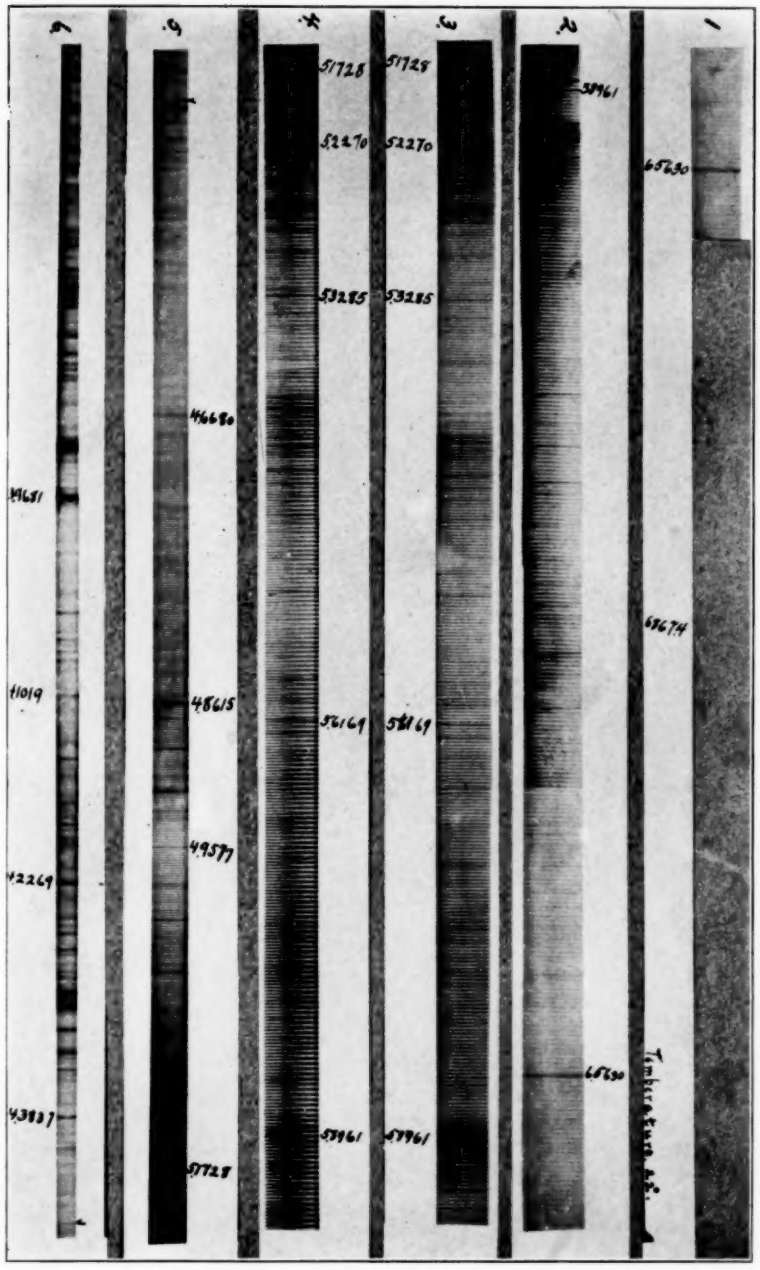

PLATE V.

S. R. WILLIAMS.

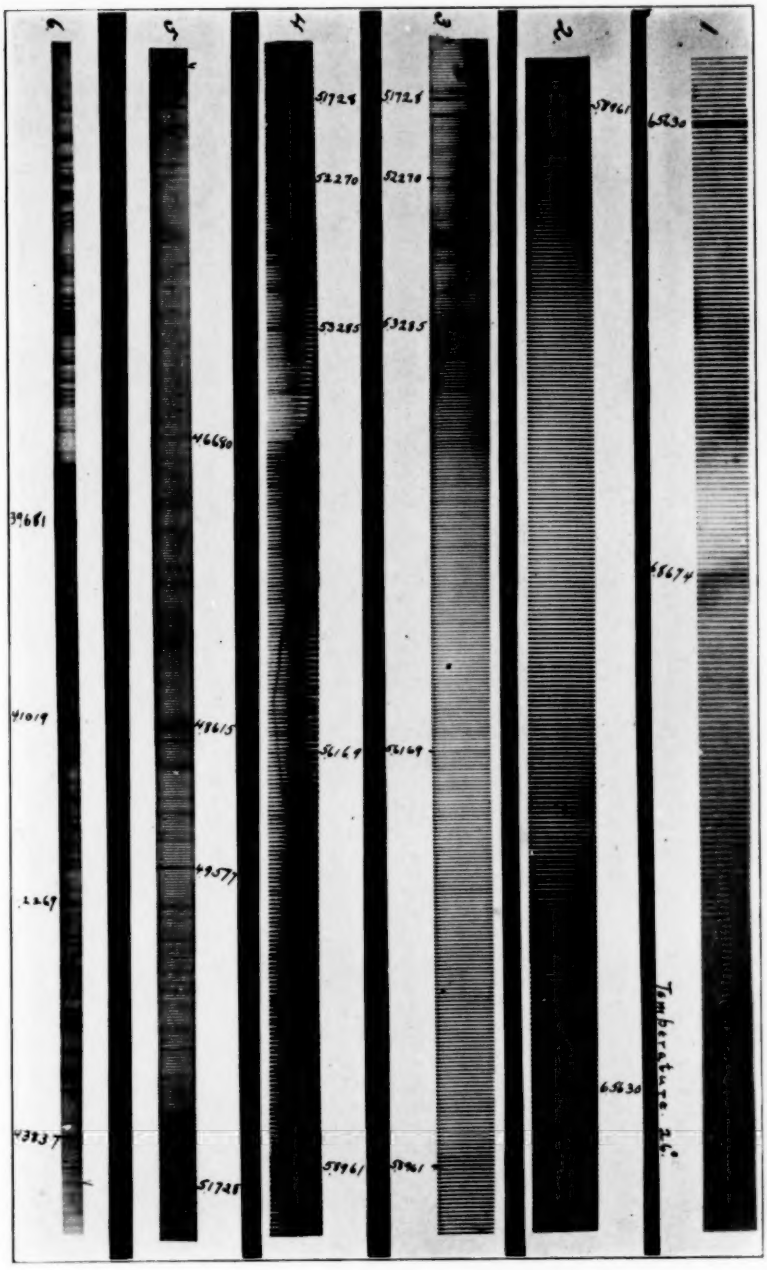

PLATE VI.

S. R. WILLIAMS.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE TWENTY-THIRD MEETING.

A REGULAR meeting of the Physical Society was held at Columbia University, New York City, on Saturday, February 27, 1904.

At the afternoon session the presidential address was delivered by President A. G. Webster, the subject being:

Some Practical Aspects of the Relations between Physics and Mathematics.

During the presidential address the American Mathematical Society met with the Physical Society in joint session.

The following papers were read:

1. The Conduction of Electricity in Mercury Vapor. A. P. WILLS.

2. Exhibition of Experiments Illustrating the Action of a Magnet on the Mercury Arc. PETER COPPER HEWITT.

3. Photography of Cloud Particles and the Photographic Record of Atmospheric Nucleation. CARL BARUS.

4. Preliminary Measurements of the Short Wave-lengths Discovered by Schumann. THEODORE LYMAN.

5. The Hall Effect in the Electric Arc. C. D. CHILD.

6. Note on Some Further Observations on the Radiation Produced in an Alternating Condenser Field. FERNANDO SANFORD.

Certain amendments to Articles I. and II. of the by-laws of the society, duly recommended by the Council, were on motion adopted. The purpose of these amendments was (1) to make it possible to elect Associate Members of the society hereafter as well as Regular Members; (2) to establish, for Regular Members, an entrance fee. Articles I. and II., as amended, read as follows:

I.

(*a*) The membership of the society shall consist of regular members, honorary members, and associate members.

(*b*) For the election of a new member to the society, either regular, honorary, or associate, or for the transfer of an associate member to

regular membership, it shall be necessary that a proposition in due form signed by two members of the society shall be presented at a meeting of the council, and that at a subsequent meeting of the council the person named in such proposition shall receive the favorable ballots of a majority of the members present.

(*c*) Associate members are entitled to all the privileges of regular members except that they may not vote nor hold office.

II.

(*a*) The annual dues of regular and associate members shall be five dollars payable on the 1st of January. Each new regular or associate member, if elected before July 1, shall pay the full dues for the year; if elected after July 1, he shall pay two dollars and a half for the half year. Should the annual dues of any member remain unpaid beyond a reasonable time, the council shall remove his name from the list of members, after due notice.

(*b*) Newly elected regular members, or members transferred from associate to regular membership, shall pay an entrance fee of three dollars.

The following announcement was made by the secretary regarding the spring meeting of the Society:

The spring meeting will be held in Washington, D. C., on Friday, April 22, and Saturday, April 23, 1904. Sessions for the presentation of papers will be held on Friday from 2 P. M. to 5 P. M., and on Saturday from 10 A. M. to 1 P. M. On Friday at 6 P. M. there will be an informal dinner, and later in the evening a lecture upon a subject to be announced later. On Saturday, at 1 P. M., a luncheon by the Philosophical Society of Washington, complimentary to the Physical Society. Saturday afternoon an excursion to the Bureau of Standards and the Weather Bureau. The arrangement of further details regarding the meeting is in the hands of a committee of the Philosophical Society of Washington, at whose invitation the meeting is held in that city.

Circulars giving further information will be mailed to all members well in advance of the meeting.

ERNEST MERRITT,
Secretary.

Some Practical Aspects of the Relations Between Physics and Mathematics.[1]

By Arthur G. Webster.

IT is now more than four years since the American Physical Society has had a presidential address, and the only one that it has had set a pitch of ideality and enthusiasm that is hardly to be expected of subsequent addresses. I shall therefore crave your indulgence if I am not able to touch so varied a field, nor to soar on eagle's pinions as did our illustrious first president on that occasion, while I invite your attention to "Some Practical Aspects of the Relations between Physics and Mathematics."

Before touching upon my specific subject, however, it is proper to briefly review some of the chief happenings in physics between that time and this. Our own society was then making its entrance into the arena, and counted a membership of only 69, while now it is a strong and flourishing organization, embracing with few exceptions every working physicist in the country, and with a membership roll of 220. This number has not been attained by making membership open to every applicant, but the attempt has been made to limit it to persons who have actually made contributions to the science. In order to extend the privileges of the society to a wider circle of those who have an interest in physics, but have not yet been able to engage in productive research, the establishment of an associate membership is now contemplated.

Our first sad duty is to take account of our losses, chief among which is that of our first president, whose many-sided experimental skill and keen insight into the secrets of nature's ways made him without a peer among us. Who among us does not feel a patriotic pride when Rowland's value of the mechanical equivalent of heat is always quoted as among the most accurate, or when we remember that for the possibility of the most accurate work on the spectrum the world was indebted to an American? The breadth of Rowland's mind and the high ideals which ever guided him in the pursuit of truth are reflected in his presidential address, which marks a red-letter day in the history of this society.

Within the year physics in this country has suffered a loss of the most grave nature. Though he was not a member of this society, in the death

[1] Presidential Address, delivered at a joint session of the American Physical Society and the American Mathematical Society, February 27, 1904.

of Josiah Willard Gibbs we all regret the loss of a master in a field to which we have as yet contributed but little to the world, the domain of theoretical or mathematical physics. To this subject Gibbs was what Rowland was to experimental physics. It is deeply to be regretted that his extreme modesty, and the very abstract nature of the subjects with which he dealt, prevented the knowledge of his work from being widely disseminated until late in his carreer, while his power to inspire and influence research might under other circumstances have been so much greater. Gibbs's chief title to fame will undoubtedly be found in his thermodynamical writings. Beginning with two important papers on geometrical methods of representation of the thermodynamical properties of bodies, by which in particular their behavior in the different states of aggregation was clearly exhibited to the eye, he showed the advantage of considering the entropy and energy of the substance as the independent variables determining its state rather than the temperature and pressure.

The laws of thermodynamics, as developed by Clausius and the other classical writers, are two: First, the principle of equivalence of work and heat which says that if a system be taken through any closed cycle of transformations of its state, the total heat given to the system $\int dQ$ is equal to the amount of work $\int dW$ done by it on external systems. If instead of a cycle we consider an infinitesimal transformation, dQ and dW are not exact differentials of any function of the variables determining the state of the system, but the principle of equivalence states that their difference is, so that

$$dQ - dW = dU, \tag{1}$$

U being a function of the state of the system, called its energy. Secondly, although dQ is not an exact differential, an integrating divisor can be found, which is the same for all substances, and, depending only on the temperature, can be taken as the absolute measure of the temperature. The function S, thus shown to exist, defined by the equation

$$dS = \frac{dQ}{\theta}, \tag{2}$$

is called the entropy, and the two laws are mathematically stated when we say that there exist an energy and an entropy function. In the definition of entropy, the transformation is supposed to be reversible, or one which is the limit approached as the conditions of equilibrium are more nearly fulfilled. In an actual transformation, less heat is transformed into work than in a reversible one, or as, for the same heat taken in, more is thrown out without being utilized, the algebraic sum of all taken in dQ is less than if the transformation were made reversibly, accordingly

$$dQ \leq \theta\, dS. \tag{3}$$

This second law of thermodynamics has been misused, especially by chemists and engineers, to prove that an infinitesimal amount of heat energy can *always* be decomposed into the product of two factors, one finite, which is here taken to be the temperature, and the other a perfect differential, as in many cases is actually true. The analogy with these cases is then supposed to demonstrate the existence of the entropy function, and to take the place of the mathematical and physical reasoning which justifies its employ. The fallacy of this reasoning appears when it is stated that in all practical transformations it is not the equality, but the inequality, that is employed.

Combining the equations of the two laws, we obtain

$$\theta\, dS \geqq dU + dW \tag{4}$$

and as soon as the work has been expressed in terms of the variables defining the state, the equation may be integrated. It may seem somewhat astonishing to the outsider that all of Gibbs's remarkable work with its extremely important consequences, many of which have been verified by experiment, while much more still awaits the experimental work which shall verify it, results from the careful consideration of this single total differential equation. Beginning with the simplest case, that of a homogeneous substance under no stress but that of a uniform pressure, the work done is given simply by the equation $dW = p dv$, so that our equation becomes

$$\theta\, dS \geqq dU + p dv. \tag{5}$$

At the same time we find that two variables, say the temperature θ and pressure p, completely determine the state of the body, so that the volume is given as a function of them. It then appears that, writing

$$dU = \theta\, dS - p dv, \tag{6}$$

the temperature and pressure are the partial derivaties of the energy by the entropy and the volume, taken positively and negatively, respectively. From this it follows that if we represent the properties of the substance by a surface whose vertical coördinate, for instance, represents the energy, while the two horizontal ones are the entropy and volume, the temperature and pressure at any state are determined by the inclination of the tangent plane, at the corresponding point, the inclination of whose intersections with two vertical planes parallel to the coördinate planes is given by the derivatives $\partial U/\partial S$, $\partial U/\partial v$. Thus from a knowledge of the energy as a function of the entropy and the volume, we obtain all the quantities with which we have to do, which is not the case if the temperature and pressure are taken for independent variables,

because then we have to integrate the differential equation (5) in order to find U and S. But not only this. By means of the sign of inequality in (5) Gibbs is able to determine conditions for the stability of the existence of a substance in a particular state, and to give it a geometrical form, which states that if a particular state is one of stable equilibrium, the representative surface at the point in question will have both its principle curvatures of the same sign, or the surface will be convexo-convex downwards, if the energy axis is drawn upwards. Maxwell was so impressed with the importance of Gibbs's researches in this line that he constructed with his own hands a model of such a surface for water in its three forms, the original of which may still be seen in the Cavendish laboratory in Cambridge, and a cast of which he sent to Professor Gibbs, who had it in his lecture-room. Before the time of Gibbs, almost the only questions of equilibrium which had been systematically dealt with had been those which treated of the equilibrium of the same substance in two states of aggregation, such as water and ice or vapor. By means of an extension of the manner of regarding the question, Gibbs was able to lay down the laws of equilibrium of substances in any number of what he called *phases;* that is, different portions each by itself homogeneous, but different one from another not only physically but even chemically. Thus the foundation was laid for a general treatment of chemical phenomena. As soon as reversible processes could be devised for the removal from a particular phase, say a salt solution, of a particular constituent, say the water, for instance by evaporation, or by freezing out, it was possible to extend the differential equation to cover the equilibrium in question. If $m_1 \cdots m_n$ denote the quantities of different chemical substances contained in a given homogeneous *phase*, the equation is written

$$dU = \theta dS - pdv + \mu_1 dm_1 + \cdots + \mu_n dm_n, \tag{7}$$

and the quantities μ_r are called by Gibbs the *potentials* relatively to this phase of the various constituents. They thus afford a direct measure of the h therto mysterious chemical affinity, and are the rates of increase of energy of the substance when unit quantity of the various substances is introduced without changing the entropy or volume of the phase. Thus to the conditions of dynamic equilibrium, that of a number of phases in presence of each other the pressure in each must be equal or there will be motion, and for thermal equilibrium that the temperature in each must be equal or there will be flow of heat, Gibbs adds the condition for chemical equilibrium, that for any constituent the chemical potential must be equal in every phase or there will be chemical reaction. As a consequence of this condition by the mere consideration of the number of variables involved, follows the celebrated *phase-rule*, which states that

in any equilibrium the number of phases present cannot be greater than two more than the number of constituents. For instance, a single constituent, like water, can be present in three phases,— solid, liquid, and vapor, — but then with no freedom, that is, only at one determined temperature and pressure, the triple point of James Thomson. If on the other hand there are only two phases, say liquid and vapor, we have one variable in terms of which the others are determined, say the temperature, which completely determines the pressure of the vapor. This phase-rule has become of extreme importance and received great experimental development at the hands of the physical chemists. Of the other subjects treated in the papers, including a complete theory of capillarity and a development of the ideas of osmotic pressure and of dilute solutions, I have not time to speak. These researches were of such an abstract character, and were expressed in such severely logical form, almost devoid of particular examples, that for years they were very little known, and being published in a journal which was little accessible, they suffered still farther, until by the efforts of the chemists Ostwald and Le Chatelier, they were made at least partially accessible in German and French translations. May I not express the hope that Yale University will now see its way to give the scientific world an edition of the complete works of Gibbs in order that we may not be longer compelled to obtain a knowledge of them through the medium of foreign languages.

It is a pleasure to know that, in spite of the unfortunate circumstances noted, the value of Gibbs's thermodynamical writings was sufficiently appreciated during his lifetime to bring him the honor of the Rumford medal of the American Academy of Arts and Sciences in 1881, and the highest distinction of the Royal Society in its Copley medal in 1901.

The bicentennial of Yale University was signalized by the publication by Professor Gibbs of a treatise in its way quite as striking as his thermodynamical researches; namely, his Elementary Principles in Statistical Mechanics. We here have a generalization of these processes which when applied to the kinetic theory of gases had been attended with such fruitful results in the hands of Maxwell and Boltzmann. Gibbs here considered the conditions of so-called statistical equilibrium of a very great number of precisely similar dynamical systems; that is, systems determined by the same number of independent variables, with the same functions giving the kinetic and potential energies in terms of the velocities and coördinates, but with the states of the different systems, as determined by the values of the coördinates and velocities at any particular instant, varying from one system to another, so as to embrace all conceivable states. The question that then presents itself is: In what manner must these states be distributed, that is, how many of each must there be, in order that as the systems move in accordance with the dynamical laws that govern

them all, the number of systems that are in any particular state shall not vary as the time goes on. Having found this condition, the next question is to decide upon the condition when the various systems are brought into relation so as to influence each other in some particular manner. To take the particularly simple example which led Maxwell to deal with the subject, consider a great number of very small material particles flying about in a given region at random; that is, with all possible velocities, and in all possible places. Supposing that all directions of the velocities are represented and that the number of particles whose velocities lie between u and $u + du$ is $\varphi(u)du$, then what must be the nature of the function φ in order that, when influencing each other by collision, or by the exercise of repulsive forces between pairs of particles, the value of φ shall be independent of the time. Maxwell found that the result was that $\varphi(u) = Au^2e^{-c^2u^2}$, or in other words that the rectangular components u_x, u_y, u_z of the velocity are distributed according to the law of probability, $e^{-c^2u_x^2}$. If there are a number of particles in statistical equilibrium it turns out that the mean kinetic energy for those of one sort is the same as for those of another sort, and this result leads us to what is known as Avogadro's law, one of the most important laws of physics for the chemist. If instead of a set of particles, each of which has three coördinates, we have a system depending on a greater number, such as a rigid body, with six, the theorem known by the name of Maxwell and Boltzman tells us that statistical equilibrium is attained when the kinetic energy is equally shared among all the different degrees of freedom for each coördinate system. For instance, if we consider a pair of material points bound together by a thin rod, forming a dumbbell-like model of a diatomic molecule, and characterized by five degrees of freedom, the mean kinetic energy of translation for all the molecules is three fifths of the whole energy, thus giving us the ratio of the specific heats equal to 1.4. These are practical conclusions of the kinetic theory of gases, and it is to the elaborate generalization of such theorems that Gibbs's work leads. The subject belongs to the mathematical theory of probability, with all the pitfalls which belong to that subject. A certain function of the state of the systems is found which, when they act upon each other, always tends to increase, and this is analogous to the thermodynamic entropy. It is in this manner that the deduction of the second law of thermodynamics, which has so long been sought as a consequence of the laws of dynamics alone, has its most promising mode of attack. Apparently this law is not to be deduced from the properties of a single system, but by averaging those of a great many systems similar to each other. It is not to be denied that if Gibbs has not succeeded in this deduction, which he modestly disclaims, he has at any rate made a great stride in the direction of success.

I can here only mention Gibbs's work on the electromagnetic theory of light, in which the same method of averages is used, and in which he makes noteworthy contributions to the difficult theory of dispersion. Finally, a few words on the subject to which Gibbs seemed to attach great importance, but which in my opinion is of far less importance than the work already mentioned. His attention was early drawn to the convenience for the physicist of the use of the notion of vectors introduced by Hamilton and Grassmann. Being dissatisfied with their notation, he devoted much attention to improving it, and made great use of it in his lectures. The great convenience of a system of vector analysis is now recognized by everyone, but how much of the short-hand advocated by one or another of its advocates is indispensible is still a matter for discussion. In my own humble opinion, what is necessary is the idea of a vector, and of the scalar or geometric, and vector products, along with that of the linear vector function and the notions of divergence and curl, while it is a matter of individual preference whether one shall write down the cartesian equations, or one of the three, or adopt a system of notation which may or may not convey more meaning to the eye, according to one's habits of mind. At any rate it may be said that, if present signs are to be trusted, there is as great a tendency to adopt the notation of Gibbs, as so well indicated and expounded in the treatise by his pupil, Dr. Wilson, as that of any of his competitors. With regard to the theory of vector analysis, considered as a branch of multiple algebra, to which Gibbs also made important contributions, I have no doubt that the pure mathematician will appreciate its value without need of the recommendation of the physicist.

I have spoken thus at length of the work of Gibbs, not only because his work was so good an illustration of my main thesis, but also because I believe that it shows him to have been possessed of one of the most commanding intellects that this country has produced, giving him the right to be ranked with Maxwell and the great names of the century now gone.

I pass now to some of the gains which physics has made in the last four years. The subject of radioactivity, which was then beginning to attract the attention of physicists, has since become of absorbing interest, and the extraordinary behavior of the element radium in the emission of energy without apparent source of supply has led us to expect developments of the greatest importance in our theoretical notions. Even the non-scientific public, ever avid of the sensational, has interested itself in the properties of radium to an extent only rivalled by the discovery of the Röntgen rays, and has apparently got the idea that radium is to undo the chief conquests in physics of the past. Of this, however, we have no fear, but we welcome the new discoveries, with the confidence

that their mysteries will eventually be unravelled and will fit themselves into our system of mechanics without rupturing it. It is a pleasure to recall that in the subject of radioactivity one of the leading contributors has been, if not an American, at any rate a professor in an institution on the American continent, a member of this society and of its council, and a frequent contributor to its meetings, Professor Rutherford, of Montreal.

During the last year of the life of Rowland, one of his earliest and most important discoveries was called in question; namely, the magnetic field produced by an electric charge in motion. It would have seemed to require a daring and experienced experimenter to question the existence of a phenomenon which had been twice verified by Rowland himself, but such was the insistence upon the claim that it could not be ignored. It is a satisfaction to know that experiments carried on under the direction of Rowland during the last months of his life had again confirmed his conclusion, and that he died in the full conviction that he had made no mistake. Others took up the subject, with equally positive results, but still the suspicion would not down. Finally, by the combination of efforts of Mr. Pender, who had verified Rowland's results in Baltimore, and of M. Crémieu, who had apparently disproved them, working together in Paris, the ghost was finally laid, let us hope not to rise again.

Within the last two years another matter of great theoretical importance has been experimentally confirmed. Maxwell, in his treatment of light as consisting of electromagnetic waves, had shown that a perfectly reflecting body exposed to light incident normally should experience a pressure numerically equal to the density of energy in the medium in front of it. When Sir Wm. Crookes discovered the action of the radiometer thirty years ago, he at first supposed that it was the pressure of light which caused the action, until it was shown that the effect was of quite a different nature, being due to the action of the residual gases on the radiometer vane. This effect of Crookes had then to be eliminated before the pressure due to the light radiation itself could be observed, and this was at last accomplished, first by the distinguished Russian physicist, Lebedew, and then, but with far greater accuracy, by our colleagues, Nichols and Hull. This brilliant confirmation of Maxwell's theoretical conclusion, involving experimentation of the highest order, constitutes in my opinion one of those achievements of which American physicists should be most proud.

The subject of mechanical flight, whether it belongs to physics or engineering, is at any rate a good example of the truth that to any engineering invention there are three stages: first, the physical, in which the laws governing the process are determined; second, the engineering,

in which it is found how they must be practically applied; and finally, the commercial, when it is found possible to put the invention into practice with profit. It is the two last stages that are most appreciated by the general public, which generally knows but little of the first. Thus the prolonged and well-planned endeavors of Langley to bring the question of flight successfully into the engineering stage, though very nearly crowned with success, have failed of popular acclaim, have been greeted with derision by a large part of the press, and made the author a target of abuse for congressmen, ignorant of the difficulty of the problem, and of the very great amount of meritorious work previously done by Professor Langley on the physical part, namely, the investigation of the laws of resistance of the air to the motion of plane surfaces. May we not, while according our admiration to these researches, also express the hope that the more practical investigations may also be brought to a successful conclusion. Pursuing the same object in a somewhat different manner, the brothers O. and W. Wright have met with even more success, and treating the problem of flight as it probably must be considered, namely as a species of athletic sport involving great skill in equilibration, and following on the lines of the unfortunate Lilienthal and later of Chanute, and learning the manipulation of soaring apparatus, have finally added mechanical propulsion, and made a successful flight of more than half a mile.

In his address, Professor Rowland lamented the lack of great endowed laboratories of research, and contrasted the amount of money devoted to the promotion of research with that devoted to armies and navies for purposes of destruction. In this connection we have some progress to report. To be sure, the United States is expecting to spend some ninety-six millions of dollars on the navy during the coming year, although not engaged in war, but the government has also established the Bureau of Standards, for the buildings and equipment of which $550,000 has been already appropriated, with a large annual appropriation including for the current year the sum of $85,000 for salaries. Under the broad policy of its director, Professor Stratton, this institution will be to a large extent a research laboratory, and plans have been made for buildings of such convenience, with so complete an equipment, that I feel confident that this laboratory will be little, if any, behind the German Physikalisch-technische Reichsanstalt, a comparison as flattering as any I know how to make. Those who saw last year at Washington what had already been begun, and the plans of the buildings now under construction, and who heard this year at St. Louis the account of the work by Professor Rosa, will, I think, agree that I have not overestimated the importance of this institution. Its work will not only influence the work of physicists in general, but will have the most advantageous in-

fluence on the work of instrument-makers in this country, and will tend to make us independent of foreign countries to an extent which has never been the case before.

Finally, as a further step in palliation of Rowland's reproach, we have the foundation of the Carnegie Institution, which to be sure has not the income of $100,000,000 per year desired by Rowland, but which is nevertheless a most generous addition to the material facilities in aid of scientific research, and which will, if wisely administered, undoubtedly have a most helpful effect in advancing the interests of science. To be sure, we find that last year the grants in physics were only five thousand dollars, while a single chemist obtained one half this sum, and for this year economic science has received a grant of thirty thousand dollars for a single object. Whether, as has been intimated to me, the representatives of physics were not sufficiently active in presenting plans of research, or whether the administrators of the Carnegie Institution had six times as much confidence in the representatives of economics as in those of physics, the result is equally unflattering to us, and we must strive to remove its cause. As I have stated above, I believe that the sending of Dr. Pender to Paris by the Carnegie Institution to investigate with M. Crémieu the Rowland effect, will, if it shall have permanently settled the existence of this effect, be of enough importance to justify the expedition of the whole sum appropriated to physics.

This is not the place for me to examine the record of this country as to its contributions to physics at large. I believe it will be generally admitted that in physics, as in other sciences, as in art and literature, our contribution has been painfully small, as compared with what might have been expected from a people as numerous, as energetic, and as prosperous as ours. The names of Franklin, Rumford, Henry, and Mayer, were the only ones mentioned by Rowland as constituting the "meager list of those whom death allows me to speak of and who have earned mention here by doing something for the progress of our science." To these is now to be added his own name, while among those still living are several whose work has won the admiration of the world. We have at any rate now the spirit, and many praiseworthy researches are here produced each year. If it is true that few capital discoveries in physics have originated in this country, to what is it due? Many replies suggest themselves. The application of so much energy to the subduing of a new country, the greater inducements offered by commercial success, the recent great material prosperity of the country, with a corresponding loss of ideality. It is undoubtedly true that the volume density of ideas connected with physics is as yet abnormally small in the atmosphere in this country, on account of the newness of the habit of inquiring of nature at first hand, and on account of the isolation of physicists from each other and from the Euro-

pean schools in which the habit of pondering on such subjects is old and well established. But I venture to propose another reason for this deficiency, not a sufficient reason, but to my mind an important one, and that is the insufficient equipment which American physicists have generally possessed in mathematics.

That this lack has been a real one hardly needs argument. Before the Civil War and even later, it was impossible to obtain instruction even in pure mathematics in anything worthy the name. The early years of the nineteenth century were great years in physical discovery, but the work of Fourier, Poisson and Cauchy, of Gauss, Green, Young, McCullagh, and Fresnel, remained a closed book in our universities. Those few rare searchers who took up investigation in physics were obliged to do so without the help that theory, embodied in mathematical symbolism, can give. But here it may be asked, in fact university presidents do ask, "Can one not become an investigator in physics without mathematics?" and the name of Faraday will be adduced as a convincing example to the contrary. To that may be replied that what was to Faraday practically a *tabula rasa* is now so thickly written over that the difficulty of making new discoveries is greatly enhanced, and the need of all possible equipment greatly emphasized; but also that in Faraday the lack of mathematical knowledge was compensated for by a most wonderful intuition, which in itself would have constituted him a genius. But even Faraday, with all his intuition, failed to show how electrical actions could be transmitted through the medium which he felt sure did transmit them, and the electromagnetic theory of light was left for another to invent. Maxwell, on the contrary, with his thorough mathematical equipment, taking up with enthusiasm the ideas of Faraday, was able to elaborate a system in which all the phenomena already known found a place, with room for the many not then known, which have since been found to justify his theory.

This example of Faraday and Maxwell may be made an excellent text for a discussion as to the object of experimental investigation in general. What is the use in observing the pressure of light? Is it merely that it is an interesting phenomenon, or is it because it fits into a great systematic arrangement of our knowledge of the mechanism of radiation, of the properties of energy and the laws of mechanics, as embodied in the differential equations representing the properties of the ether. Is there, I ask, a grander achievement of the human intellect than this same theory of the ether, not built up like the theories of the ancients on *a priori* grounds and hypotheses, but arrived at by a gradual process of logical reasoning as more and more experimental facts became known, and gradually taking on a simplified and comprehensible form, so that in the words of Lord Kelvin, ether is the only thing we know anything about. Why are we so interested in the confirmation of Rowland's observation that an

electrified disk in rotation produces a magnetic field? Would that fact of itself be so important if it did not fit in with Maxwell's theory that charges are found at regions of divergence of the electric polarization of the medium, and that where this electric polarization is changing with the time, there is produced a magnetic field. Thus the phenomenon of Rowland takes its place as furnishing a term in the equations of Maxwell, explicitly introduced into them by Hertz, and contributing toward their symmetry and consistency. And if the Rowland effect does not exist, what becomes of the Zeeman effect, and of the determination of the mass of electrons by the curvature of cathode rays in a magnetic field?

But, it may be objected, the content of a theory, such as the theory of the ether, may be expressed in physical language without the employment of mathematical symbols and differential equations. Faraday's lines of force and his electrotonic state of the medium, it may be said, express as much as the triple integrals and the time derivatives of the mathematician. To a certain extent this may be true, but this extent must be very limited, and although, for instance, Faraday was able to state the law of induction of currents quantitatively in terms of the number of lines of force threading a circuit, without reference to a surface integral expressing the flux, he was not able to state the quantitative laws of distribution of energy in the medium, to describe the method of propagation of the energy in waves, or to foresee the mechanical pressure of the radiation.

Besides the argument resulting from the desire to coördinate our facts in a systematic theory, let us consider the practical help which mathematics affords to the experimental investigator as to what to look for. In the address from which I have so often quoted, Professor Rowland makes use of the statement that "a mathematical investigation always obeys the law of the conservation of knowledge: we never get out more from it than we put in." This when taken literally is to be sure true, but it would be a great mistake to infer therefrom the practical fruitlessness of mathematical investigation of natural phenomena. Consider the possibilities of transformation. Let us not forget that although chemical action obeys the laws of conservation of energy, still from the mixture of the sour nitric acid and the sweet and harmless glycerine we get the powerful nitro-glycerine, fraught with powers for good or evil. So in the hands of Maxwell the volume integrals of Gauss and Laplace representing action at a distance are transformed into differential equations representing action of the medium and propagation in time. This result is then left for experimentation to confirm, as it did in such a satisfactory manner at the hands of Hertz. As a further example, let us consider the position of an investigator of the conduction of heat who should be ignorant of the work of Fourier. To be sure, he might find out the laws of the stationary state, and this knowledge might stand to what we now

have of conduction in the relation of the knowledge of steady electric currents to that of all the phenomena of variable and periodic currents of electricity. The laws of heat conduction are in themselves extremely simple, and are embodied in the law of Fourier that the quantity of heat flowing across an isothermal surface in a given small interval of time is proportional to the duration of the time, to the area of the surface, and, most important, to the rate of fall of temperature in the direction of the normal to the isothermal surface. This law of conduction together with the previously known one that the elevation of temperature produced in a substance by the communication of a quantity of heat is proportional to that quantity, leads to the equation for the propagation of heat by conduction,

$$\frac{\partial u}{\partial t} = a^2 \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right\}.$$

In like manner our knowledge of the propagation of a sound wave consists in the two statements that the acceleration imparted to a small portion of the medium is proportional to the resultant of all the pressures acting on it, and that the component in any direction is proportional to the rate of decrease of pressure as we go in that direction, and secondly that the pressure produced at a point depends in a definite manner on the compression produced there, say according to the adiabatic law. These two statements again lead us to the differential equation for the propagation of sound, which is, what is not the least interesting, the same as that for the propagation of light and of electric waves, as the one just mentioned is also applicable to the diffusion of liquids or gases,

$$\frac{\partial^2 u}{\partial t^2} = a^2 \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right\}.$$

Is there now, I may ask, anything in the nature of the physical laws involved or in the manner of stating them which would lead one to infer that there was any extraordinary difference in the two modes of propagation? Is there even any such marked difference in the differential equations? I think that you will agree with me that it would take a physicist of very great intuition to describe exactly what would occur in the two cases, if, for instance, a small region in an unlimited medium were to be the seat of an initial disturbance, in the one case representing a hot spot, in the other a region of compression, or electric or magnetic field. And yet the beautiful method invented by Fourier and developed by Cauchy leads to the integral of the conduction equation, in case the temperature is at a given time, $t = 0$, distributed in any prescribed manner $u = f(xyz)$, so that the temperature at any subsequent time t is represented by the integral

$$u = \frac{1}{a^3\pi^{\frac{3}{2}}} \int\int\int_{-\infty}^{\infty} f(\alpha,\beta,\gamma) e^{-r^2/4a^2t} t^{-\frac{3}{2}} d\alpha d\beta d\gamma,$$

where r represents the distance of the point $\alpha\beta\gamma$ from the point xyz. A glance at this integral tells the whole story, as follows:

The temperature of any point xyz is the additive resultant of the effects of all the initially heated points, each producing an effect dependent on its distance away, and on the time that has elapsed, in accordance with the factor $e^{-r^2/4a^2t}/t^{\frac{3}{2}}$ which, on discussion as a function of the time t and the distance r, gives the following results. The effect at the point xyz, no matter how far away, is *immediately* felt, the temperature rises to a certain maximum and then falls away, but the effect is never lost. The maximum temperature attained is less the greater the distance between the two points, and the rate of rise and fall is slower, but the striking thing is that some effect is immediately produced, as shown graphically by the curves which I show. Nor is the effect of the distance to be overlooked, for although the time of maximum temperature is greater the greater the distance, it is not proportional to the distance, but to its square. Thus we see how futile it would be to seek a velocity of propagation of heat, for there is none. This solution, when applied by Lord Kelvin to the propagation of electric current in a cable in which the effect of self-induction was neglected, showing that the maximum of signals would be received at times proportional to the squares of the distances, gave rise to the famous KR-law; and it was upon this basis that Lord Kelvin predicted the success of the Atlantic cable, by comparisons with the working of a short Mediterranean cable already existing. Here, then, was a practical result of a mathematical nature readily appreciated by capitalists of the caliber of Cyrus W. Field.

Contrast the result just described with the result of the same method applied to the second equation. Without writing down the result, or the transformation which enables us to interpret it, which is longer, we find that the effect at any point is the resultant of the conditions at other points, but that in this case the effect is not immediately felt, but that it comes on at a time directly proportional to the distance between the two points, passes over, and the effect is gone. Thus sound, light and electrical disturbances are propagated with a definite velocity in waves, and we are able at all instants to identify the separate effects of each point, or, as we may say, to interpret signals sent us without confusion with each other. It was by the discussion of the two terms in the integral of this equation that Kirchoff put the principle of Huyghens, and the explanation of interference and diffraction, upon a satisfactory dynamical basis.

In connection with the two differential equations which I have men-

tioned there comes the one which is an extension of them, and which arises when we attempt to find the law of propagation of sound in a viscous medium, of electric waves in a conducting dielectric, or of a telegraph line whose self-induction is not negligible. This gives the equation, first obtained by Kirchhoff,

$$\frac{\partial^2 u}{\partial t^2} + 2b\frac{\partial u}{\partial t} = a^2\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right\}$$

which has been called the telegraphist's equation, and which is of so much importance practically in connection with telephony and the transmission of energy by alternating currents. It may here be guessed with physical intuition that the physical nature of the propagation will combine the characters of both the other modes, and this the mathematical discussion will verify. Here the method of Fourier is again applicable, although this was long unperceived, and the labors of Heaviside, Picard, Poincaré, Boussinesq, and Brillouin have at last given us the general solution, which shows that while the propagation is by waves, and with a definite velocity, still the wave is in a measure wiped out as it proceeds, and leaves behind it a persistent trail which lasts indefinitely, and in telegraphy confuses the signals. The discussion of the solution, however, suggested to Vaschy and Heaviside what was to be done practically to improve the transmission, and render telephony possible over still greater distances. It was the results of a mathematical discussion of particular solutions of the telegraphist's equation, in a paper read in this room, that led Professor Pupin to the great improvement in telephony which he has so successfully completed. It is of the greatest importance that the method of Riemann, and the applications of it by Darboux, Picard and others, have enabled us to classify equations so that the matter of the propagation of discontinuities stands out in clear relief.

Let us now turn to the questions of the future or of the immediate present. What are they? Though the ether itself is now well understood, at least as to its mode of action, the nature of its connection with matter is now prominently under discussion, and the electro-dynamics of moving bodies is an extremely important subject for investigation. The importance of small charged bodies or electrons in the subject of radioactivity has aroused a very great interest in the question of the magnetic fields produced by moving charges, and every result of theory will be eagerly welcomed. Here the theory of Lorentz seems to be of the greatest promise, being an adaptation of Maxwell's theory to moving bodies, and the results of this theory have even led to the attempt on the part of Wien, Abraham, and others to explain dynamics itself on an electromagnetic basis, the inertia of a particle being due to the reaction of the magnetic field emitted by a moving electric charge (the Rowland

effect). The question of the relative motion of the earth and the ether, with the explanation of aberration is occupying the attention of the most skilled talent, both experimentally and theoretically. The question of the mechanics of radiation, and the distribution of energy in the spectra of hot bodies is another subject of the highest importance, in which great progress has recently been made, and more is confidently expected. Besides these, there is the ever-present attempt to simplify matters by a reduction to dynamical explanations, particularly in the matter of thermodynamics, where the statistical methods of which I have spoken are undoubtedly to become of still greater importance. All of these subjects demand mathematical acumen and equipment of a high order, and it seems needless to argue the absolute necessity of such equipment to the successful physicist wishing to contribute in large degree to the extension of the world's knowledge. To the objection sometimes made, that the two kinds of mind, which are fitted at the same time for experimental discovery and for mathematical research, are rarely combined in the same person, and the suggesstion that the physicist ally himself with some mathematician who shall solve his theoretical difficulties, I reply that such happy marriages are rarer still, and that lacking an extraordinary amount of sympathy and insight on the part of the mathematician, the advice of Miles Standish is still good,

> "If you wish a thing to be well done,
> You must do it yourself, you must not leave it to others."

As examples to illustrate my point, I may mention the names of Helmholtz, Lord Rayleigh, and Hertz, of whom it would be difficult to say whether they excelled most in theory or experiment.

We come then to the practical question, how much and what portions of mathematics are necessary to the physicist, and how may the student be provided with them. It is unnecessary to say that first of all comes a thorough acquaintance with the infinitesimal calculus, with the ability to make use of it in manifold applications. Of these, the most important is mechanics, the foundation stone of physics, and the portion which we wish to include everything else. Upon a thorough training in mechanics one cannot too strongly insist. And here it will not suffice to be content with the elementary notions, but the student must understand and be practised in the meaning and use of Lagrange's equations, and be acquainted with the methods used in the treatment of continuous media, as well as of discrete points. The subject of differential equations is of fundamental importance in all physical applications, and that of linear partial differential equations with constant coefficients should receive particular attention. As an indication of what is desirable in this direction I may mention Byerly's treatise on Fourier's series and Weber's reconstituted lectures of Riemann, of which the first may be especially

commended for the many examples for practice furnished the student, and the latter for the greater variety of methods with which it gives acquaintance. The subject of the theory of functions of a complex variable is of such manifold importance in its physical applications, especially in connection with the conformal representation of surfaces, and with the linear differential equations with variable coefficients that arise from physical problems, that it must not be neglected. I believe also that it is impossible for the physicist to be too familiar with geometry, and in particular with those applications of the calculus usually denoted by the name differential geometry, as applied to the study of curves and surfaces.

Having arrived at this point, I take the liberty of addressing myself to the mathematicians, and contemplating briefly the other side of the shield, by showing what they are to gain from the alliance with the physicists. I need hardly recapitulate the debt which mathematics owes to physical questions for proposing fruitful fields of inquiry. There is no need to go back to ancient times to speak of the measurement of land as giving the impetus to the study of geometry, or even to the time of Newton, whose attention was chiefly occupied by physical questions. From the time of Laplace and Lagrange, the greatest mathematical intellects have concerned themselves with physical matters. Gauss, who by common consent is without a peer in his century, was the author not only of the "Disquisitiones Arithmeticæ" but also of the "Forces acting according to the inverse square." To how many investigations in the theory of differential equations and the theory of functions has the equation of Laplace given rise! Was not the investigation of Dirichlet on the convergence of trigonometric series, brought to the attention of the world by their application by Fourier to physical problems, the beginning of the modern critical study of functions of a real variable, and has not the subject thus begun been one of the most prolific in all modern analysis? Did not the differential equations which have been most thoroughly studied, namely the linear differential equations of the second order, originate from the method of orthogonal curvilinear coördinates, applied by Lamé to the study of the partial differential equations of physics? Was not the subject of the Calculus of Variations called into existence, and does it not still draw most of its problems or examples from physical questions? I will mention only brachistochrones and minimal surfaces. Even such exponents of the "purest" or least applied species of mathematics as Weierstrass and Kronecker found an application of the theory of elementary divisors in Lagrange's problem of small oscillations, while the attention paid to physical applications by Poincaré, Picard and Klein is well known. But there is a still more practical reason that the mathematician should concern himself with physics, to a degree that is by no means uncommon in Europe, but is I believe far from common in this

country. I may say in fact, to speak in language that is sure to be understood in America, that it will be money in the pocket of the mathematician to make himself acquainted to a large extent with physics, and with the needs of physicists and even of engineers. Many a promising student, who has earned his doctor's degree in pure mathematics finds himself obliged to teach applications to physics, and what is likely to be his state of mind, however familiar he may be with the theory of groups or with the properties of power series, if he knows nothing of the potential function, of dynamics, or of spherical harmonics? I have even known mathematical graduates to have to teach thermodynamics and the strength of materials, and in engineering schools this is by no means infrequently the case.

I hope then that I have said enough to show that the interests of mathematicians and physicists are to a large degree identical, and that each can well profit by the work of the other. At any rate, they are both equally interested that the instruction in mathematics in the schools should be improved, and that throughout school and college the most practical methods should be adopted, with the minimum of waste. In this matter I am thoroughly in accord with the ideas expressed by Professor Moore in his presidential address to the American Mathematical Society a year ago, particularly in what he says with regard to the unification of pure and applied mathematics. The complaint is often made of the instruction in the schools that the algebra, and even the geometry, is largely meaningless to the scholar because he does not have any use for it. Now mathematics, though it has undoubtedly its *raison d'être* if regarded simply as an intellectual achievement, is for the great majority of those who use it a tool, to be mastered because one can do something with it, and the sooner that the student learns what he can do with it the better. I believe then that with every step in advance in algebra or geometry should be connected some application, so that the drawing-board, the slide-rule and the performance of experiments in physics should all be closely bound together. In fact I would go so far as to wish that all the school mathematics and physics might be taught by the same person, or if this is not possible, that the work of the two departments should be in the hands of teachers in the highest sympathy with each other's work. In this regard I share the desires of Professor John Perry in England, who is continually contending for the extension of the teaching of the useful and interesting rather than that of the formal or strictly logical.

I shall now attempt briefly to sketch out a course, such as I think a student of physics should have, and to show that it is quite practical, and that without making inordinate demands. I claim no originality, as such courses are in existence at many of our leading institutions — I wish

merely to urge that all intending physicists take them. I will suppose that the school course in geometry, algebra and physics has been organized as described by Professor Moore, and that the boy comes to college familiar with the elements of algebra, including the complex diagram, skilled with the drawing-board, the ruler and compasses, and practised in numerical computations, the use of tables and squared paper. He should also know enough of plane trigonometry to get along with — it is astonishing how little that is.

Most of our colleges now have four year courses, and a students spends in addition three in graduate work before taking his degree. It will not be long before the college course is reduced to three years, whether we approve or no, and while it is not possible to make a physicist or mathematician in six years, we can at least give him a good start, and launch him on the scientific sea prepared to navigate for himself, without expecting him to make shipwreck. During the first year he will naturally devote himself to analytic geometry, of two and three dimensions, for I believe that soon after beginning they may be studied together, and to learning enough of the elements of the theory of equations and of determinants to apply them in geometry. By the end of this year he should have been introduced to the elements of calculus, without which I do not believe it pays to study analytic geometry. In connection with the analytic geometry he may also learn the elements of vector analysis, which is largely a matter of notions and notations. If anyone doubt the possibility of doing this in a year's course of five hours a week, I recommend him to read the preface of Perry's calculus, and see what that gifted teacher has himself accomplished. During the second year he should attack the calculus in a serious manner, guided by a text-book covering the subject matter of, say, Lamb's admirable text-book, learning at the same time more analytic geometry, and supplementing the examples with many from different parts of physics, such as are bountifully supplied by Perry. During this year he will certainly have made the acquaintance of mechanics, in a more satisfactory way than was possible the year before, and by the end of the year will have a good deal of practical knowledge of differential equations. Of course the squared paper and the drawing-board will have been in constant use. The third year he will get at partial derivatives with a multitude of examples, definite integrals, an extremely practical subject for the physicist, and differential geometry. To give a concrete idea of what I mean, I will suppose the subject matter to have been covered in these three years that is contained in Lamb's and Perry's Calculus, Appell's Elements de l'Analyse mathématique (an extremely attractive and practical book), and perhaps half of Scheffers' Anwendung der Differential- und Integralrechnung auf Geometrie. The student will then be prepared to take up mathematical physics in

earnest, and during the next three years the systematic study of differential equations occurring in physics, with the developments in series of functions of Fourier, Bessel, Laplace and Lamé, the theory of functions and of infinite series and the mathematical theories of electricity and light, sound and heat, all preceded by a liberal amount of mechanics, including elasticity and hydrodynamics, will have no terrors for him. It may be asked whether I would require all these subjects for the doctor's degree. I reply that I certainly should, if by that degree is meant an adequate preparation for life, understanding by "life" in this case the contribution of original work to the world.

In the college work I believe that it would be of mutual advantage not to separate the two classes of students, mathematical and physical, nor them from those who pursue the subjects merely for their educational value, but in the university practical requirements may require a modification of this opinion. Here, again, I appeal to my mathematical colleagues that they have the greatest sympathy for the needs of physical and engineering students who may come under their teaching. Remember that though they, too, may have good brains and appreciate accurate logic, still to them mathematics is a thing to do something with, and that they require more than proofs that things are so, the knowledge that they may be used. Do not concentrate your attention upon existence theorems, or upon the question of the convergence of series to the exclusion of what I may call the probable conditions of convergence. On this point I may call attention to the views of Professor Klein in his last book, On the Application of the Calculus to Geometry, and to his remarks on approximation-mathematics. Of course, it is necessary to know what uniform convergence is, and that it is not always possible to differentiate or integrate a series term by term, but surely the instruction should lay more force on "thou shalt" than on "thou shalt not."

A friend of mine describes the effect on students of the teaching of one of his ablest colleagues, by saying that they are so scared that they never dare to integrate, differentiate, or touch an infinite series, for fear that something awful may happen. How different is this from the methods of Fourier, Green, Maxwell, and more recently of Heaviside, who, all untrammeled by academical restrictions, proceeds with the most extraordinary facility to make new contributions to our knowledge, leaving to other less original minds the task of demonstrating the correctness of his methods.

Let no one say that it is impossible to steer the difficult course between the Scylla of rigor and the Charybdis of inaccuracy, for it has been done by excellent professional mathematicians in a way to be both attractive and profitable to the physicist. I need only instance the admirable Analytisch-Funktiontheoretische Vorlesungen of Fricke (whose Differ-

ential Calculus I would also commend in the same breath as that of Perry, though a great contrast to it) and in the Partielle Differentialgleichungen of Weber, whose rewriting of Riemann's course has maintained the clearness and elegance it had, while adding immensely to it in variety. A course on these two books would constitute a liberal education in mathematics for the physical student, practical in the highest degree.

I have now sketched in a rough way what I think we can do for the aspiring physicist in the way of providing him with a mathematical panoply, and that without preventing him from spending the proper amount of time in the laboratory and of devoting a reasonable amount of time to the study of English, French or German, or even Latin and Greek. Fifteen hours per week is not too much to spend in the classroom while in college, and I believe that if the physical or mathematical student divides this equally between physics, mathematics and general culture, he will be safe. To those who demand that all the time in college should be devoted to general culture and none to preparation for the business of life, I have only to say that they do not understand the exigencies of the strenuous life of scientific research. I regret, as much as they do, that there should be any physicists ignorant of Greek, to say nothing of Latin. What little of them I know I would not exchange for any number of definite integrals, or even milligrams of radium. Culture and the love of the beautiful and of the ideal we must all have — the more the better ; but let it be understood that this may come in different ways, and that we cannot all read the same books or think the same thoughts. Let not the poet, the musician and the painter set themselves over against us, however, and rail at us as devoted to the material. If they do so, they do it in ignorance, and let us not antagonize them, for we strive for the same ends. Ours is as idealistic a pursuit as that of the poet, and calls for as much imagination. It is an intellectual joy that we strive after, which nature affords to us as well. Is the sky less blue to us because we can specify its wave lengths, or because we know that its color is due to diffraction by small particles? Is the music less sweet because we can resolve its motion into harmonics and develop it in a series of normal functions? I think not. It seemed to me particularly appropriate to see a fine pipe-organ in the drawing room of a great physicist, equally noted for his mathematical and experimental contributions. Far be it from me to wish to substitute for Southey's description of " How the Water Comes Down at Lodore " —

> " Thumping and plumping and bumping and jumping,
> And dashing, and flashing, and splashing and clashing,
> And so never ending, but always descending,
> Sounds and motions for ever and ever are blending,
> All and at once and all o'er, with a mighty uproar —
> And this way the water comes down at Lodore," —

the statement that it descends with a uniform acceleration of 980 centimeters per second per second, modified by the effect of viscosity and impact, and subject to the equation of continuity, that the vapor tension about drops of spray is larger than over smooth water, and that the region about is abnormally defective in radioactivity. Both sorts of statement have their place, and both appeal to the imagination. There are times when fact is of the most consequence; other times when fancy. The lines of Tennyson,

"Every moment dies a man,
Every moment one is born,"

are surely not improved by Babbage's suggested emendation in the interest of accuracy,

"Every moment dies a man,
And one and a sixteenth is born."

But facts lead to fancies, and in the quest of nature's secrets, we are lifted above the material philistinism of every-day life.

May the Americal Physical Society, and the American Mathematical Society ever continue their ways, like twin sisters, hand in hand, in pursuit of the good, the true and the beautiful.

The Conduction of Electricity in Mercury Vapor.[1]

By A. P. Will.

IF X represent the electromotive intensity in volts in the positive column of a tube containing saturated mercury vapor carrying a current of electricity, then

$$X = f(I, \Delta, D)$$

wherein I represents the current in amperes, Δ the vapor pressure in millimeters of mercury, D the diameter of the tube in centimeters. An approximate empirical form for the function f was determined; also the numerical values of the constants involved were found. This was accomplished by making experiments upon tubes of different diameters (varying from 1.25 cm. to 5.08 cm.) with different currents (varying from .5 of an ampere to 3 amperes) at different temperatures, care being taken that the measurements were made when the vapor was saturated. Thus the vapor pressure and the vapor density could be deduced. Two thermo-couples were introduced into a tube about midway of its length at a distance from each other of about 15 centimeters. These served at once as a means of determining the temperature and the drop in potential, the latter being measured by a Kelvin electrostatic voltmeter.

It was found that for a constant current and a constant diameter the relation between X and Δ is linear up to a certain pressure Δ_Q while for higher values of the pressure the relation though still linear is represented by a line of lesser slope. The pressure Δ_Q depends upon both the current and the diameter.

The approximate form found for f is

$$X = a\left\{b + \frac{1}{\sqrt{I}}\right\}\left\{c + \frac{1}{\sqrt{D}}\right\}\left\{\Delta + \frac{1}{\sqrt{D}} + \frac{2}{D}\right\} + a,$$

wherein a, b, c, and a are constants which suddenly change in value when the pressure Δ_Q is reached; the numerical values of the constants which do not change appear in the equation. The numerical values of the constants which jump in value are:

	Below Δ_Q	Above Δ_Q
a	.055	.15
b	.775	.398
c	1.71	.122
a	.10	.37

[1] Abstract of a paper presented at the meeting of the Physical Society held on February 27, 1900.

It was difficult to determine Δ_Q as a function of the current and the diameter very accurately but a rough approximation is

$$\Delta_Q = K \frac{\sqrt[4]{I}}{\sqrt{D}},$$

wherein $K = 4$.

The main part of the discussion was with reference to the positive column. But the matter of the drop in potential over the anode and over the cathode was also touched upon. The potential drop over both anode and cathode depends but slightly upon the current. Over the anode the drop in potential is about 7 volts and over the cathode about 5 volts. Under abnormal conditions, for instance when the anode is covered with a bluish, velvety haze the potential drop over it may rise to 15 volts or more.

The experiments were carried out in the laboratory of Dr. Peter Cooper Hewitt, without whose collaboration the work would have been impossible.

PRELIMINARY MEASUREMENT OF THE SHORT WAVE-LENGTHS DISCOVERED BY SCHUMANN.[1]

BY THEODORE LYMAN.

FOR the past few years the author has been engaged in an attempt to measure the short wave-length discovered by Dr. Victor Schumann, but it is only recently that this attempt has proved successful.

Working in an atmosphere of hydrogen with a concave diffraction grating ruled upon speculum metal, an "end-on" tube filled with hydrogen gives numerous lines below the aluminum group at 1854.

The shortest wave-length so far observed has a value of 1178 Ångström units. The limit of error is two units.

It is interesting to note that, contrary to expectation, speculum metal is able to reflect these very short wave-lengths to a considerable degree.

This is but a preliminary notice. The author has in preparation a complete list of these new wave-lengths. He has also good hopes of still further extending the spectrum.

It is the author's purpose to publish a detailed account of the investigation in the Proceedings of the American Academy, and perhaps elsewhere.

[1] Abstract of a paper presented at a meeting of the Physical Society held on February 27, 1904.